

### III.

## THE GENESIS OF SCIENCE.\*

---

**T**HERE has ever prevailed among men a vague notion that scientific knowledge differs in nature from ordinary knowledge. By the Greeks, with whom Mathematics—literally *things learnt*—was alone considered as knowledge proper, the distinction must have been strongly felt; and it has ever since maintained itself in the general mind. Though, considering the contrast between the achievements of science and those of daily unmethodic thinking, it is not surprising that such a distinction has been assumed; yet it needs but to rise a little above the common point of view, to see that no such distinction can really exist: or that at best, it is but a superficial distinction. The same faculties are employed in both cases; and in both cases their mode of operation is fundamentally the same.

If we say that science is organized knowledge, we are met by the truth that all knowledge is organized in a greater or less degree—that the commonest actions of the household and the field presuppose facts colligated, inferences drawn, results expected; and that the general success of these actions proves the data by which they were guided to have been correctly put together. If, again, we say that science is prevision—is a seeing beforehand—is a know-

ing in what times, places, combinations, or sequences, specified phenomena will be found ; we are yet obliged to confess that the definition includes much that is utterly foreign to science in its ordinary acceptation. For example, a child's knowledge of an apple. This, as far as it goes consists in previsions. When a child sees a certain form and colours, it knows that if it puts out its hand it will have certain impressions of resistance, and roundness, and smoothness ; and if it bites, a certain taste. And manifestly its general acquaintance with surrounding objects is of like nature—is made up of facts concerning them, so grouped as that any part of a group being perceived, the existence of the other facts included in it is foreseen.

If, once more, we say that science is *exact* prevision, we still fail to establish the supposed difference. Not only do we find that much of what we call science is not exact, and that some of it, as physiology, can never become exact ; but we find further, that many of the previsions constituting the common stock alike of wise and ignorant, *are exact*. That an unsupported body will fall ; that a lighted candle will go out when immersed in water ; that ice will melt when thrown on the fire—these, and many like predictions relating to the familiar properties of things have as high a degree of accuracy as predictions are capable of. It is true that the results predicated are of a very general character ; but it is none the less true that they are rigorously correct as far as they go : and this is all that is requisite to fulfil the definition. There is perfect accordance between the anticipated phenomena and the actual ones ; and no more than this can be said of the highest achievements of the sciences specially characterised as exact.

Seeing thus that the assumed distinction between scientific knowledge and common knowledge is not logically justifiable ; and yet feeling, as we must, that however impossible it may be to draw a line between them, the two

are not practically identical; there arises the question—What is the relationship that exists between them? A partial answer to this question may be drawn from the illustrations just given. On reconsidering them, it will be observed that those portions of ordinary knowledge which are identical in character with scientific knowledge, comprehend only such combinations of phenomena as are directly cognizable by the senses, and are of simple, invariable nature. That the smoke from a fire which she is lighting will ascend, and that the fire will presently boil water, are previsions which the servant-girl makes equally well with the most learned physicist; they are equally certain, equally exact with his; but they are previsions concerning phenomena in constant and direct relation—phenomena that follow visibly and immediately after their antecedents—phenomena of which the causation is neither remote nor obscure—phenomena which may be predicted by the simplest possible act of reasoning.

If, now, we pass to the previsions constituting what is commonly known as science—that an eclipse of the moon will happen at a specified time; and when a barometer is taken to the top of a mountain of known height, the mercurial column will descend a stated number of inches; that the poles of a galvanic battery immersed in water will give off, the one an inflammable and the other an inflaming gas, in definite ratio—we perceive that the relations involved are not of a kind habitually presented to our senses; that they depend, some of them, upon special combinations of causes; and that in some of them the connection between antecedents and consequents is established only by an elaborate series of inferences. The broad distinction, therefore, between the two orders of knowledge, is not in their nature, but in their remoteness from perception.

If we regard the cases in their most general aspect, we see that the labourer, who, on hearing certain notes in the

adjacent hedge, can describe the particular form and colours of the bird making them; and the astronomer, who, having calculated a transit of Venus, can delineate the black spot entering on the sun's disc, as it will appear through the telescope, at a specified hour; do essentially the same thing. Each knows that on fulfilling the requisite conditions, he shall have a preconceived impression—that after a definite series of actions will come a group of sensations of a foreknown kind. The difference, then, is not in the fundamental character of the mental acts; or in the correctness of the previsions accomplished by them; but in the complexity of the processes required to achieve the previsions. Much of our commonest knowledge is, as far as it goes, rigorously precise. Science does not increase this precision; cannot transcend it. What then does it do? It reduces other knowledge to the same degree of precision. That certainty which direct perception gives us respecting coexistences and sequences of the simplest and most accessible kind, science gives us respecting coexistences and sequences, complex in their dependencies or inaccessible to immediate observation. In brief, regarded from this point of view, science may be called *an extension of the perceptions by means of reasoning*.

On further considering the matter, however, it will perhaps be felt that this definition does not express the whole fact—that inseparable as science may be from common knowledge, and completely as we may fill up the gap between the simplest previsions of the child and the most recondite ones of the natural philosopher, by interposing a series of previsions in which the complexity of reasoning involved is greater and greater, there is yet a difference between the two beyond that which is here described. And this is true. But the difference is still not such as enables us to draw the assumed line of demarcation. It is a difference not between common knowledge and scientific knowl-

edge ; but between the successive phases of science itself, or knowledge itself—whichever we choose to call it. In its earlier phases science attains only to *certainty* of foreknowledge ; in its later phases it further attains to *completeness*. We begin by discovering *a* relation : we end by discovering *the* relation. Our first achievement is to foretell the *kind* of phenomenon which will occur under specific conditions : our last achievement is to foretell not only the kind but the *amount*. Or, to reduce the proposition to its most definite form—undeveloped science is *qualitative* prevision : developed science is *quantitative* prevision.

This will at once be perceived to express the remaining distinction between the lower and the higher stages of positive knowledge. The prediction that a piece of lead will take more force to lift it than a piece of wood of equal size, exhibits certainty, but not completeness, of foresight. The kind of effect in which the one body will exceed the other is foreseen ; but not the amount by which it will exceed. There is qualitative prevision only. On the other hand, the prediction that at a stated time two particular planets will be in conjunction ; that by means of a lever having arms in a given ratio, a known force will raise just so many pounds ; that to decompose a specified quantity of sulphate of iron by carbonate of soda will require so many grains—these predictions exhibit foreknowledge, not only of the nature of the effects to be produced, but of the magnitude, either of the effects themselves, of the agencies producing them, or of the distance in time or space at which they will be produced. There is not only qualitative but quantitative prevision.

And this is the unexpressed difference which leads us to consider certain orders of knowledge as especially scientific when contrasted with knowledge in general. Are the phenomena *measurable* ? is the test which we unconsciously

employ. Space is measurable: hence Geometry. Force and space are measurable: hence Statics. Time, force, and space are measurable: hence Dynamics. The invention of the barometer enabled men to extend the principles of mechanics to the atmosphere; and Aerostatics existed. When a thermometer was devised there arose a science of heat, which was before impossible. Such of our sensations as we have not yet found modes of measuring do not originate sciences. We have no science of smells; nor have we one of tastes. We have a science of the relations of sounds differing in pitch, because we have discovered a way to measure them; but we have no science of sounds in respect to their loudness or their *timbre*, because we have got no measures of loudness and *timbre*.

Obviously it is this reduction of the sensible phenomena it represents, to relations of magnitude, which gives to any division of knowledge its especially scientific character. Originally men's knowledge of weights and forces was in the same condition as their knowledge of smells and tastes is now—a knowledge not extending beyond that given by the unaided sensations; and it remained so until weighing instruments and dynamometers were invented. Before there were hour-glasses and olepsydras, most phenomena could be estimated as to their durations and intervals, with no greater precision than degrees of hardness can be estimated by the fingers. Until a thermometric scale was contrived, men's judgments respecting relative amounts of heat stood on the same footing with their present judgments respecting relative amounts of sound. And as in these initial stages, with no aids to observation, only the roughest comparisons of cases could be made, and only the most marked differences perceived; it is obvious that only the most simple laws of dependence could be ascertained—only those laws which being uncomplicated with others, and not disturbed in their manifestations, required no nice-

ties of observation to disentangle them. Whence it appears not only that in proportion as knowledge becomes quantitative do its previsions become complete as well as certain, but that until its assumption of a quantitative character it is necessarily confined to the most elementary relations.

Moreover it is to be remarked that while, on the one hand, we can discover the laws of the greater proportion of phenomena only by investigating them quantitatively; on the other hand we can extend the range of our quantitative previsions only as fast as we detect the laws of the results we predict. For clearly the ability to specify the magnitude of a result inaccessible to direct measurement, implies knowledge of its mode of dependence on something which can be measured—implies that we know the particular fact dealt with to be an instance of some more general fact. Thus the extent to which our quantitative previsions have been carried in any direction, indicates the depth to which our knowledge reaches in that direction. And here, as another aspect of the same fact, we may further observe that as we pass from qualitative to quantitative prevision, we pass from inductive science to deductive science. Science while purely inductive is purely qualitative: when inaccurately quantitative it usually consists of part induction, part deduction: and it becomes accurately quantitative only when wholly deductive. We do not mean that the deductive and the quantitative are coextensive; for there is manifestly much deduction that is qualitative only. We mean that all quantitative prevision is reached deductively; and that induction can achieve only qualitative prevision.

Still, however, it must not be supposed that these distinctions enable us to separate ordinary knowledge from science; much as they seem to do so. While they show in what consists the broad contrast between the extreme forms of the two, they yet lead us to recognise their essential iden-

tity ; and once more prove the difference to be one of degree only. For, on the one hand, the commonest positive knowledge is to some extent quantitative ; seeing that the amount of the foreseen result is known within certain wide limits. And, on the other hand, the highest quantitative prevision does not reach the exact truth, but only a very near approximation to it. Without clocks the savage knows that the day is longer in the summer than in the winter ; without scales he knows that stone is heavier than flesh : that is, he can foresee respecting certain results that their amounts will exceed these, and be less than those—he knows *about* what they will be. And, with his most delicate instruments and most elaborate calculations, all that the man of science can do, is to reduce the difference between the foreseen and the actual results to an unimportant quantity.

Moreover, it must be borne in mind not only that all the sciences are qualitative in their first stages,—not only that some of them, as Chemistry, have but recently reached the quantitative stage—but that the most advanced sciences have attained to their present power of determining quantities not present to the senses, or not directly measurable, by a slow process of improvement extending through thousands of years. So that science and the knowledge of the uncultured are alike in the nature of their previsions, widely as they differ in range ; they possess a common imperfection, though this is immensely greater in the last than in the first ; and the transition from the one to the other has been through a series of steps by which the imperfection has been rendered continually less, and the range continually wider.

These facts, that science and the positive knowledge of the uncultured cannot be separated in nature, and that the one is but a perfected and extended form of the other, must necessarily underlie the whole theory of science, its

progress, and the relations of its parts to each other. There must be serious incompleteness in any history of the sciences, which, leaving out of view the first steps of their genesis, commences with them only when they assume definite forms. There must be grave defects, if not a general untruth, in a philosophy of the sciences considered in their interdependence and development, which neglects the inquiry how they came to be distinct sciences, and how they were severally evolved out of the chaos of primitive ideas.

Not only a direct consideration of the matter, but all analogy, goes to show that in the earlier and simpler stages must be sought the key to all subsequent intricacies. The time was when the anatomy and physiology of the human being were studied by themselves—when the adult man was analyzed and the relations of parts and of functions investigated, without reference either to the relations exhibited in the embryo or to the homologous relations existing in other creatures. Now, however, it has become manifest that no true conceptions, no true generalizations, are possible under such conditions. Anatomists and physiologists now find that the real natures of organs and tissues can be ascertained only by tracing their early evolution; and that the affinities between existing genera can be satisfactorily made out only by examining the fossil genera to which they are allied. Well, is it not clear that the like must be true concerning all things that undergo development? Is not science a growth? Has not science, too, its embryology? And must not the neglect of its embryology lead to a misunderstanding of the principles of its evolution and of its existing organization?

There are *à priori* reasons, therefore, for doubting the truth of all philosophies of the sciences which tacitly proceed upon the common notion that scientific knowledge and ordinary knowledge are separate; instead of commencing, as they should, by affiliating the one upon the

other, and showing how it gradually came to be distinguishable from the other. We may expect to find their generalizations essentially artificial; and we shall not be deceived. Some illustrations of this may here be fitly introduced, by way of preliminary to a brief sketch of the genesis of science from the point of view indicated. And we cannot more readily find such illustrations than by glancing at a few of the various *classifications* of the sciences that have from time to time been proposed. To consider all of them would take too much space: we must content ourselves with some of the latest.

Commencing with those which may be soonest disposed of, let us notice first the arrangement propounded by Oken. An abstract of it runs thus:—

Part I. MATHESIS.—*Pneumatogeny*: Primary Art, Primary Consciousness, God, Primary Rest, Time, Polarity, Motion, Man, Space, Point, Line, Surface, Globe, Rotation.—*Hylogeny*: Gravity, Matter, Ether, Heavenly Bodies, Light, Heat, Fire.

(He explains that MATHESIS is the doctrine of the whole; *Pneumatogeny* being the doctrine of immaterial totalities, and *Hylogeny* that of material totalities.)

Part II. ONTOLOGY.—*Cosmogeny*: Rest, Centre, Motion, Line, Planets, Form, Planetary System, Comets.—*Stöchiogeny*: Condensation, Simple Matter, Elements, Air, Water, Earth.—*Stöchiology*: Functions of the Elements, &c. &c.—*Kingdoms of Nature*: Individuals.

(He says in explanation that "ONTOLOGY teaches us the phenomena of matter. The first of these are the heavenly bodies comprehended by *Cosmogeny*. These divide into elements—*Stöchiogeny*. The earth element divides into minerals—*Mineralogy*. These unite into one collective body—*Geogeny*. The whole in singulars is the living, or *Organic*,

which again divides into plants and animals. *Biology*, therefore, divides into *Organogeny*, *Phytosophy*, *Zoosophy*.”)

FIRST KINGDOM.—MINERALS. *Mineralogy*, *Geology*.

Part III. BIOLOGY.—*Organosophy*, *Phytogeny*, *Phyto-physiology*, *Phytology*, *Zoogeny*, *Physiology*, *Zoology*, *Psychology*.’

A glance over this confused scheme shows that it is an attempt to classify knowledge, not after the order in which it has been, or may be, built up in the human consciousness; but after an assumed order of creation. It is a pseudo-scientific cosmogony, akin to those which men have enunciated from the earliest times downwards; and only a little more respectable. As such it will not be thought worthy of much consideration by those who, like ourselves, hold that experience is the sole origin of knowledge. Otherwise, it might have been needful to dwell on the incongruities of the arrangements—to ask how motion can be treated of before space? how there can be rotation without matter to rotate? how polarity can be dealt with without involving points and lines? But it will serve our present purpose just to point out a few of the extreme absurdities resulting from the doctrine which Oken seems to hold in common with Hegel, that “to philosophize on Nature is to re-think the great thought of Creation.” Here is a sample:—

“Mathematics is the universal science; so also is Physio-philosophy, although it is only a part, or rather but a condition of the universe; both are one, or mutually congruent.

“Mathematics is, however, a science of mere forms without substance. Physio-philosophy is, therefore, *mathematics endowed with substance*.”

From the English point of view it is sufficiently amusing to find such a dogma not only gravely stated, but stated as an unquestionable truth. Here we see the expe-

riences of quantitative relations which men have gathered from surrounding bodies and generalized (experiences which had been scarcely at all generalized at the beginning of the historic period)—we find these generalized experiences, these intellectual abstractions, elevated into concrete actualities, projected back into Nature, and considered as the internal frame-work of things—the skeleton by which matter is sustained. But this new form of the old realism, is by no means the most startling of the physiophilosophic principles. We presently read that,

“The highest mathematical idea, or the fundamental principle of all mathematics is the zero = 0.” \* \* \*

“Zero is in itself nothing. Mathematics is based upon nothing, and, *consequently*, arises out of nothing.

“Out of nothing, *therefore*, it is possible for something to arise; for mathematics, consisting of propositions, is something, in relation to 0.”

By such “consequentlys” and “therefores” it is, that men philosophize when they “re-think the great thought of creation.” By dogmas that pretend to be reasons, nothing is made to generate mathematics; and by clothing mathematics with matter, we have the universe! If now we deny, as we *do* deny, that the highest mathematical idea is the zero;—if, on the other hand, we assert, as we *do* assert, that the fundamental idea underlying all mathematics, is that of equality; the whole of Oken’s cosmogony disappears. And here, indeed, we may see illustrated, the distinctive peculiarity of the German method of procedure in these matters—the bastard *à priori* method, as it may be termed. The legitimate *à priori* method sets out with propositions of which the negation is inconceivable; the *à priori* method as illegitimately applied, sets out either with propositions of which the negation is *not* inconceivable, or with propositions like Oken’s, of which the *affirmation* is inconceivable.

It is needless to proceed further with the analysis; else might we detail the steps by which Oken arrives at the conclusions that “the planets are coagulated colours, for they are coagulated light; that the sphere is the expanded nothing;” that gravity is “a weighty nothing, a heavy essence, striving towards a centre;” that “the earth is the identical, water the indifferent, air the different; or the first the centre, the second the radius, the last the periphery of the general globe or of fire.” To comment on them would be nearly as absurd as are the propositions themselves. Let us pass on to another of the German systems of knowledge—that of Hegel.

The simple fact that Hegel puts Jacob Bœhme on a par with Bacon, suffices alone to show that his stand-point is far remote from the one usually regarded as scientific: so far remote, indeed, that it is not easy to find any common basis on which to found a criticism. Those who hold that the mind is moulded into conformity with surrounding things by the agency of surrounding things, are necessarily at a loss how to deal with those, who, like Schelling and Hegel, assert that surrounding things are solidified mind—that Nature is “petrified intelligence.” However, let us briefly glance at Hegel’s classification. He divides philosophy into three parts:—

1. *Logic*, or the science of the idea in itself, the pure idea.

2. *The Philosophy of Nature*, or the science of the idea considered under its other form—of the idea as Nature.

3. *The Philosophy of the Mind*, or the science of the idea in its return to itself.

Of these, the second is divided into the natural sciences, commonly so called; so that in its more detailed form the series runs thus:—Logic, Mechanics, Physics, Organic Physics, Psychology.

Now, if we believe with Hegel, first, that thought is the

true essence of man ; second, that thought is the essence of the world ; and that, therefore, there is nothing but thought ; his classification, beginning with the science of pure thought, may be acceptable. But otherwise, it is an obvious objection to his arrangement, that thought implies things thought of—that there can be no logical forms without the substance of experience—that the science of ideas and the science of things must have a simultaneous origin. Hegel, however, anticipates this objection, and, in his obstinate idealism, replies, that the contrary is true ; that all contained in the forms, to become something, requires to be thought : and that logical forms are the foundations of all things.

It is not surprising that, starting from such premises, and reasoning after this fashion, Hegel finds his way to strange conclusions. Out of *space* and *time* he proceeds to build up *motion, matter, repulsion, attraction, weight, and inertia*. He then goes on to logically evolve the solar system. In doing this he widely diverges from the Newtonian theory ; reaches by syllogism the conviction that the planets are the most perfect celestial bodies ; and, not being able to bring the stars within his theory, says that they are mere formal existences and not living matter, and that as compared with the solar system they are as little admirable as a cutaneous eruption or a swarm of flies.\*

Results so outrageous might be left as self-disproved, were it not that speculators of this class are not alarmed by any amount of incongruity with established beliefs. The only efficient mode of treating systems like this of Hegel, is to show that they are self-destructive—that by their first steps they ignore that authority on which all their subsequent steps depend. If Hegel professes, as he manifestly does, to develop his scheme by reasoning—if he presents

\* It is somewhat curious that the author of "The Plurality of Worlds," with quite other aims, should have persuaded himself into similar conclusions.

successive inferences as *necessarily following* from certain premises; he implies the postulate that a belief which necessarily follows after certain antecedents is a true belief: and, did an opponent reply to one of his inferences, that, though it was impossible to think the opposite, yet the opposite was true, he would consider the reply irrational. The procedure, however, which he would thus condemn as destructive of all thinking whatever, is just the procedure exhibited in the enunciation of his own first principles.

Mankind find themselves unable to conceive that there can be thought without things thought of. Hegel, however, asserts that there *can* be thought without things thought of. That ultimate test of a true proposition—the inability of the human mind to conceive the negation of it—which in all other cases he considers valid, he considers invalid where it suits his convenience to do so; and yet at the same time denies the right of an opponent to follow his example. If it is competent for him to posit dogmas, which are the direct negations of what human consciousness recognises; then is it also competent for his antagonists to stop him at every step in his argument by saying, that though the particular inference he is drawing seems to his mind, and to all minds, necessarily to follow from the premises, yet it is not true, but the contrary inference is true. Or, to state the dilemma in another form:—If he sets out with inconceivable propositions, then may he with equal propriety make all his succeeding propositions inconceivable ones—may at every step throughout his reasoning draw exactly the opposite conclusion to that which seems involved.

Hegel's mode of procedure being thus essentially suicidal, the Hegelian classification which depends upon it, falls to the ground. Let us consider next that of M. Comte.

As all his readers must admit, M. Comte presents us with a scheme of the sciences which, unlike the foregoing

ones, demands respectful consideration. Widely as we differ from him, we cheerfully bear witness to the largeness of his views, the clearness of his reasoning, and the value of his speculations as contributing to intellectual progress. Did we believe a serial arrangement of the sciences to be possible, that of M. Comte would certainly be the one we should adopt. His fundamental propositions are thoroughly intelligible; and if not true, have a great semblance of truth. His successive steps are logically co-ordinated; and he supports his conclusions by a considerable amount of evidence—evidence which, so long as it is not critically examined, or not met by counter evidence, seems to substantiate his positions. But it only needs to assume that antagonistic attitude which *ought* to be assumed towards new doctrines, in the belief that, if true, they will prosper by conquering objectors—it needs but to test his leading doctrines either by other facts than those he cites, or by his own facts differently applied, to at once show that they will not stand. We will proceed thus to deal with the general principle on which he bases his hierarchy of the sciences.

In the second chapter of his *Cours de Philosophie Positive*, M. Comte says:—"Our problem is, then, to find the one *rational* order, amongst a host of possible systems." . . . "This order is determined by the degree of simplicity, or, what comes to the same thing, of generality of their phenomena." And the arrangement he deduces runs thus: *Mathematics, Astronomy, Physics, Chemistry, Physiology, Social Physics*. This he asserts to be "the true *filiation* of the sciences." He asserts further, that the principle of progression from a greater to a less degree of generality, "which gives this order to the whole body of science, arranges the parts of each science." And, finally, he asserts that the gradations thus established *a priori* among the sciences, and the parts of each science, "is

in essential conformity with the order which has spontaneously taken place among the branches of natural philosophy ;" or, in other words—corresponds with the order of historic development.

Let us compare these assertions with the facts. That there may be perfect fairness, let us make no choice, but take as the field for our comparison, the succeeding section treating of the first science—Mathematics; and let us use none but M. Comte's own facts, and his own admissions. Confining ourselves to this one science, of course our comparisons must be between its several parts. M. Comte says, that the parts of each science must be arranged in the order of their decreasing generality; and that this order of decreasing generality agrees with the order of historic development. Our inquiry must be, then, whether the history of mathematics confirms this statement.

Carrying out his principle, M. Comte divides Mathematics into "Abstract Mathematics, or the Calculus (taking the word in its most extended sense) and Concrete Mathematics, which is composed of General Geometry and of Rational Mechanics." The subject-matter of the first of these is *number*; the subject-matter of the second includes *space, time, motion, force*. The one possesses the highest possible degree of generality; for all things whatever admit of enumeration. The others are less general; seeing that there are endless phenomena that are not cognizable either by general geometry or rational mechanics. In conformity with the alleged law, therefore, the evolution of the calculus must throughout have preceded the evolution of the concrete sub-sciences. Now somewhat awkwardly for him, the first remark M. Comte makes bearing upon this point is, that "from an historical point of view, mathematical analysis *appears to have risen out of* the contemplation of geometrical and mechanical facts." True, he goes on to say that, "it is not the less independent of

these sciences logically speaking ;” for that “analytical ideas are, above all others, universal, abstract, and simple and geometrical conceptions are necessarily founded on them.”

We will not take advantage of this last passage to charge M. Comte with teaching, after the fashion of Hegel, that there can be thought without things thought of. We are content simply to compare the two assertions, that analysis arose out of the contemplation of geometrical and mechanical facts, and that geometrical conceptions are founded upon analytical ones. Literally interpreted they exactly cancel each other. Interpreted, however, in a liberal sense, they imply, what we believe to be demonstrable, that the two had *a simultaneous origin*. The passage is either nonsense, or it is an admission that abstract and concrete mathematics are coeval. Thus, at the very first step, the alleged congruity between the order of generality and the order of evolution, does not hold good.

But may it not be that though abstract and concrete mathematics took their rise at the same time, the one afterwards developed more rapidly than the other ; and has ever since remained in advance of it ? No : and again we call M. Comte himself as witness. Fortunately for his argument he has said nothing respecting the early stages of the concrete and abstract divisions after their divergence from a common root ; otherwise the advent of Algebra long after the Greek geometry had reached a high development, would have been an inconvenient fact for him to deal with. But passing over this, and limiting ourselves to his own statements, we find, at the opening of the next chapter, the admission, that “the historical development of the abstract portion of mathematical science has, since the time of Descartes, been for the most part *determined* by that of the concrete.” Further on we read

respecting algebraic functions that "most functions were concrete in their origin—even those which are at present the most purely abstract; and the ancients discovered only through geometrical definitions elementary algebraic properties of functions to which a numerical value was not attached till long afterwards, rendering abstract to us what was concrete to the old geometers." How do these statements tally with his doctrine? Again, having divided the calculus into algebraic and arithmetical, M. Comte admits, as perforce he must, that the algebraic is more general than the arithmetical; yet he will not say that algebra preceded arithmetic in point of time. And again, having divided the calculus of functions into the calculus of direct functions (common algebra) and the calculus of indirect functions (transcendental analysis), he is obliged to speak of this last as possessing a higher generality than the first; yet it is far more modern. Indeed, by implication, M. Comte himself confesses this incongruity; for he says:—"It might seem that the transcendental analysis ought to be studied before the ordinary, as it provides the equations which the other has to resolve; but though the transcendental *is logically independent of the ordinary*, it is best to follow the usual method of study, taking the ordinary first." In all these cases, then, as well as at the close of the section where he predicts that mathematicians will in time "create procedures of *a wider generality*," M. Comte makes admissions that are diametrically opposed to the alleged law.

In the succeeding chapters treating of the concrete department of mathematics, we find similar contradictions. M. Comte himself names the geometry of the ancients *special geometry*, and that of moderns the *general geometry*. He admits that while "the ancients studied geometry with reference to the *bodies* under notice, or specially; the moderns study it with reference to the *phenomena* to be

considered, or generally." He admits that while "the ancients extracted all they could out of one line or surface before passing to another," "the moderns, since Descartes, employ themselves on questions which relate to any figure whatever." These facts are the reverse of what, according to his theory, they should be. So, too, in mechanics. Before dividing it into statics and dynamics, M. Comte treats of the three laws of *motion*, and is obliged to do so; for statics, the more *general* of the two divisions, though it does not involve motion, is impossible as a science until the laws of motion are ascertained. Yet the laws of motion pertain to dynamics, the more *special* of the divisions. Further on he points out that after Archimedes, who discovered the law of equilibrium of the lever, statics made no progress until the establishment of dynamics enabled us to seek "the conditions of equilibrium through the laws of the composition of forces." And he adds—"At this day *this is the method universally employed*. At the first glance it does not appear the most rational—dynamics being more complicated than statics, and precedence being natural to the simpler. It would, in fact, be more philosophical to refer dynamics to statics, as has since been done." Sundry discoveries are afterwards detailed, showing how completely the development of statics has been achieved by considering its problems dynamically; and before the close of the section M. Comte remarks that "before hydrostatics could be comprehended under statics, it was necessary that the abstract theory of equilibrium should be made so general as to apply directly to fluids as well as solids. This was accomplished when Lagrange supplied, as the basis of the whole of rational mechanics, the single principle of virtual velocities." In which statement we have two facts directly at variance with M. Comte's doctrine;—first, that the simpler science, statics, reached its present development only by the aid of the principle of virtual velocities, which be-

longs to the more complex science, dynamics ; and that this "single principle" underlying all rational mechanics—this *most general form* which includes alike the relations of statical, hydrostatical, and dynamical forces—was reached so late as the time of Lagrange.

Thus it is *not* true that the historical succession of the divisions of mathematics has corresponded with the order of decreasing generality. It is *not* true that abstract mathematics was evolved antecedently to, and independently of concrete mathematics. It is *not* true that of the subdivisions of abstract mathematics, the more general came before the more special. And it is *not* true that concrete mathematics, in either of its two sections, began with the most abstract and advanced to the less abstract truths.

It may be well to mention, parenthetically, that in defending his alleged law of progression from the general to the special, M. Comte somewhere comments upon the two meanings of the word *general*, and the resulting liability to confusion. Without now discussing whether the asserted distinction can be maintained in other cases, it is manifest that it does not exist here. In sundry of the instances above quoted, the endeavors made by M. Comte himself to disguise, or to explain away, the precedence of the special over the general, clearly indicate that the generality spoken of, is of the kind meant by his formula. And it needs but a brief consideration of the matter to show that, even did he attempt it, he could not distinguish this generality, which, as above proved, frequently comes last, from the generality which he says always comes first. For what is the nature of that mental process by which objects, dimensions, weights, times, and the rest, are found capable of having their relations expressed numerically? It is the formation of certain abstract conceptions of unity, duality and multiplicity, which are applicable to all things alike. It is the invention of general symbols serving to express the numer-

ical relations of entities, whatever be their special characters. And what is the nature of the mental process by which numbers are found capable of having their relations expressed algebraically? It is just the same. It is the formation of certain abstract conceptions of numerical functions which are the same whatever be the magnitudes of the numbers. It is the invention of general symbols serving to express the relations between numbers, as numbers express the relations between things. And transcendental analysis stands to algebra in the same position that algebra stands in to arithmetic.

To briefly illustrate their respective powers;—arithmetic can express in one formula the value of a *particular* tangent to a *particular* curve; algebra can express in one formula the values of *all* tangents to a *particular* curve; transcendental analysis can express in one formula the values of *all* tangents to *all* curves. Just as arithmetic deals with the common properties of lines, areas, bulks, forces, periods; so does algebra deal with the common properties of the numbers which arithmetic presents; so does transcendental analysis deal with the common properties of the equations exhibited by algebra. Thus, the generality of the higher branches of the calculus, when compared with the lower, is the same kind of generality as that of the lower branches when compared with geometry or mechanics. And on examination it will be found that the like relation exists in the various other cases above given.

Having shown that M. Comte's alleged law of progression does not hold among the several parts of the same science, let us see how it agrees with the facts when applied to separate sciences. "Astronomy," says M. Comte, at the opening of Book III., "was a positive science, in its geometrical aspect, from the earliest days of the school of Alexandria; but Physics, which we are now to consider, had no positive character at all till Galileo made his great discov-

eries on the fall of heavy bodies." On this, our comment is simply that it is a misrepresentation based upon an arbitrary misuse of words—a mere verbal artifice. By choosing to exclude from terrestrial physics those laws of magnitude, motion, and position, which he includes in celestial physics, M. Comte makes it appear that the one owes nothing to the other. Not only is this altogether unwarrantable, but it is radically inconsistent with his own scheme of divisions. At the outset he says—and as the point is important we quote from the original—"Pour la *physique inorganique* nous voyons d'abord, en nous conformant toujours à l'ordre de généralité et de dépendance des phénomènes, qu'elle doit être partagée en deux sections distinctes, suivant qu'elle considère les phénomènes généraux de l'univers, ou, en particulier, ceux que présentent les corps terrestres. D'où la physique céleste, ou l'astronomie, soit géométrique, soit mécanique ; et la physique terrestre."

Here then we have *inorganic physics* clearly divided into *celestial physics* and *terrestrial physics*—the phenomena presented by the universe, and the phenomena presented by earthly bodies. If now celestial bodies and terrestrial bodies exhibit sundry leading phenomena in common, as they do, how can the generalization of these common phenomena be considered as pertaining to the one class rather than to the other? If inorganic physics includes geometry (which M. Comte has made it do by comprehending *geometrical astronomy* in its sub-section—celestial physics); and if its sub-section—terrestrial physics, treats of things having geometrical properties; how can the laws of geometrical relations be excluded from terrestrial physics? Clearly if celestial physics includes the geometry of objects in the heavens, terrestrial physics includes the geometry of objects on the earth. And if terrestrial physics includes terrestrial geometry, while celestial physics includes celestial geometry, then the geometrical part of terrestrial physics

precedes the geometrical part of celestial physics; seeing that geometry gained its first ideas from surrounding objects. Until men had learnt geometrical relations from bodies on the earth, it was impossible for them to understand the geometrical relations of bodies in the heavens.

So, too, with celestial mechanics, which had terrestrial mechanics for its parent. The very conception of *force*, which underlies the whole of mechanical astronomy, is borrowed from our earthly experiences; and the leading laws of mechanical action as exhibited in scales, levers, projectiles, &c., had to be ascertained before the dynamics of the solar system could be entered upon. What were the laws made use of by Newton in working out his grand discovery? The law of falling bodies disclosed by Galileo; that of the composition of forces also disclosed by Galileo; and that of centrifugal force found out by Huyghens—all of them generalizations of terrestrial physics. Yet, with facts like these before him, M. Comte places astronomy before physics in order of evolution! He does not compare the geometrical parts of the two together, and the mechanical parts of the two together; for this would by no means suit his hypothesis. But he compares the geometrical part of the one with the mechanical part of the other, and so gives a semblance of truth to his position. He is led away by a verbal delusion. Had he confined his attention to the things and disregarded the words, he would have seen that before mankind scientifically co-ordinated *any one class of phenomena* displayed in the heavens, they had previously co-ordinated a *parallel class of phenomena* displayed upon the surface of the earth.

Were it needful we could fill a score pages with the incongruities of M. Comte's scheme. But the foregoing samples will suffice. So far is his law of evolution of the sciences from being tenable, that, by following his example, and arbitrarily ignoring one class of facts, it would be

possible to present, with great plausibility, just the opposite generalization to that which he enunciates. While he asserts that the rational order of the sciences, like the order of their historic development, "is determined by the degree of simplicity, or, what comes to the same thing, of generality of their phenomena;" it might contrariwise be asserted, that, commencing with the complex and the special, mankind have progressed step by step to a knowledge of greater simplicity and wider generality. So much evidence is there of this as to have drawn from Whewell, in his *History of the Inductive Sciences*, the general remark that "the reader has already seen repeatedly in the course of this history, complex and derivative principles presenting themselves to men's minds before simple and elementary ones."

Even from M. Comte's own work, numerous facts, admissions, and arguments, might be picked out, tending to show this. We have already quoted his words in proof that both abstract and concrete mathematics have progressed towards a higher degree of generality, and that he looks forward to a higher generality still. Just to strengthen this adverse hypothesis, let us take a further instance. From the *particular* case of the scales, the law of equilibrium of which was familiar to the earliest nations known, Archimedes advanced to the more *general* case of the unequal lever with unequal weights; the law of equilibrium of which *includes* that of the scales. By the help of Galileo's discovery concerning the composition of forces, D'Alembert "established, for the first time, the equations of equilibrium of *any* system of forces applied to the different points of a solid body"—equations which include all cases of levers and an infinity of cases besides. Clearly this is progress towards a higher generality—towards a knowledge more independent of special circumstances—towards a study of phenomena "the most disengaged from the incidents of

particular cases;" which is M. Comte's definition of "the most simple phenomena." Does it not indeed follow from the familiarly admitted fact, that mental advance is from the concrete to the abstract, from the particular to the general, that the universal and therefore most simple truths are the last to be discovered? Is not the government of the solar system by a force varying inversely as the square of the distance, a simpler conception than any that preceded it? Should we ever succeed in reducing all orders of phenomena to some single law—say of atomic action, as M. Comte suggests—must not that law answer to his test of being *independent* of all others, and therefore most simple? And would not such a law generalize the phenomena of gravity, cohesion, atomic affinity, and electric repulsion, just as the laws of number generalize the quantitative phenomena of space, time and force?

The possibility of saying so much in support of an hypothesis the very reverse of M. Comte's, at once proves that his generalization is only a half-truth. The fact is, that neither proposition is correct by itself; and the actuality is expressed only by putting the two together. The progress of science is duplex: it is at once from the special to the general, and from the general to the special: it is analytical and synthetical at the same time.

M. Comte himself observes that the evolution of science has been accomplished by the division of labour; but he quite misstates the mode in which this division of labour has operated. As he describes it, it has simply been an arrangement of phenomena into classes, and the study of each class by itself. He does not recognise the constant effect of progress in each class upon *all* other classes; but only on the class succeeding it in his hierarchical scale. Or if he occasionally admits collateral influences and intercommunications, he does it so grudgingly, and so quickly puts the admissions out of sight and forgets them, as to leave the

impression that, with but trifling exceptions, the sciences aid each other only in the order of their alleged succession. The fact is, however, that the division of labour in science, like the division of labour in society, and like the "physiological division of labour" in individual organisms, has been not only a specialization of functions, but a continuous helping of each division by all the others, and of all by each. Every particular class of inquirers has, as it were, secreted its own particular order of truths from the general mass of material which observation accumulates; and all other classes of inquirers have made use of these truths as fast as they were elaborated, with the effect of enabling them the better to elaborate each its own order of truths.

It was thus in sundry of the cases we have quoted as at variance with M. Comte's doctrine. It was thus with the application of Huyghens's optical discovery to astronomical observation by Galileo. It was thus with the application of the isochronism of the pendulum to the making of instruments for measuring intervals, astronomical and other. It was thus when the discovery that the refraction and dispersion of light did not follow the same law of variation, affected both astronomy and physiology by giving us achromatic telescopes and microscopes. It was thus when Bradley's discovery of the aberration of light enabled him to make the first step towards ascertaining the motions of the stars. It was thus when Cavendish's torsion-balance experiment determined the specific gravity of the earth, and so gave a datum for calculating the specific gravities of the sun and planets. It was thus when tables of atmospheric refraction enabled observers to write down the real places of the heavenly bodies instead of their apparent places. It was thus when the discovery of the different expansibilities of metals by heat, gave us the means of correcting our chronometrical measurements of astronomical periods. It was thus when the lines of the prismatic spectrum were

used to distinguish the heavenly bodies that are of like nature with the sun from those which are not. It was thus when, as recently, an electro-telegraphic instrument was invented for the more accurate registration of meridional transits. It was thus when the difference in the rates of a clock at the equator, and nearer the poles, gave data for calculating the oblateness of the earth, and accounting for the precession of the equinoxes. It was thus—but it is needless to continue.

Here, within our own limited knowledge of its history, we have named ten additional cases in which the single science of astronomy has owed its advance to sciences coming *after* it in M. Comte's series. Not only its secondary steps, but its greatest revolutions have been thus determined. Kepler could not have discovered his celebrated laws had it not been for Tycho Brahe's accurate observations; and it was only after some progress in physical and chemical science that the improved instruments with which those observations were made, became possible. The heliocentric theory of the solar system had to wait until the invention of the telescope before it could be finally established. Nay, even the grand discovery of all—the law of gravitation—depended for its proof upon an operation of physical science, the measurement of a degree on the Earth's surface. So completely indeed did it thus depend, that Newton *had actually abandoned his hypothesis* because the length of a degree, as then stated, brought out wrong results; and it was only after Picart's more exact measurement was published, that he returned to his calculations and proved his great generalization. Now this constant intercommunion, which, for brevity's sake, we have illustrated in the case of one science only, has been taking place with all the sciences. Throughout the whole course of their evolution there has been a continuous *consensus* of the sciences—a *consensus* exhibiting a general correspondence with the *consensus* of facul.

ties in each phase of mental development; the one being an objective registry of the subjective state of the other.

From our present point of view, then, it becomes obvious that the conception of a *serial* arrangement of the sciences is a vicious one. It is not simply that the schemes we have examined are untenable; but it is that the sciences cannot be rightly placed in any linear order whatever. It is not simply that, as M. Comte admits, a classification "will always involve something, if not arbitrary, at least artificial;" it is not, as he would have us believe, that, neglecting minor imperfections a classification may be substantially true; but it is that any grouping of the sciences in a succession gives a radically erroneous idea of their genesis and their dependencies. There is no "one *rational* order among a host of possible systems." There is no "true *filiation* of the sciences." The whole hypothesis is fundamentally false. Indeed, it needs but a glance at its origin to see at once how baseless it is. Why a *series*? What reason have we to suppose that the sciences admit of a *linear* arrangement? Where is our warrant for assuming that there is some *succession* in which they can be placed? There is no reason; no warrant. Whence then has arisen the supposition? To use M. Comte's own phraseology, we should say, it is a metaphysical conception. It adds another to the cases constantly occurring, of the human mind being made the measure of Nature. We are obliged to think in sequence; it is the law of our minds that we must consider subjects separately, one after another: *therefore* Nature must be serial—*therefore* the sciences must be classifiable in a succession. See here the birth of the notion, and the sole evidence of its truth. Men have been obliged when arranging in books their schemes of education and systems of knowledge, to choose *some* order or other. And from inquiring what is the best

order, have naturally fallen into the belief that there is an order which truly represents the facts—have persevered in seeking such an order; quite overlooking the previous question whether it is likely that Nature has consulted the convenience of book-making.

For, German philosophers, who hold that Nature is “petrified intelligence,” and that logical forms are the foundations of all things, it is a consistent hypothesis that as thought is serial, Nature is serial; but that M. Comte, who is so bitter an opponent of all anthropomorphism, even in its most evanescent shapes, should have committed the mistake of imposing upon the external world an arrangement which so obviously springs from a limitation of the human consciousness, is somewhat strange. And it is the more strange when we call to mind how, at the outset, M. Comte remarks that in the beginning “*toutes les sciences sont cultivées simultanément par les mêmes esprits* ;” this is “*inevitable et même indispensable* ;” and how he further remarks that the different sciences are “*comme les diverses branches d’un tronc unique*.” Were it not accounted for by the distorting influence of a cherished hypothesis, it would be scarcely possible to understand how, after recognising truths like these, M. Comte should have persisted in attempting to construct “*une échelle encyclopédique*.”

The metaphor which M. Comte has here so inconsistently used to express the relations of the sciences—branches of one trunk—is an approximation to the truth, though not the truth itself. It suggests the facts that the sciences had a common origin; that they have been developing simultaneously; and that they have been from time to time dividing and sub-dividing. But it does not suggest the yet more important fact, that the divisions and sub-divisions thus arising do not remain separate, but now and again re-unite in direct and indirect ways. They

mosculate; they severally send off and receive connecting growths; and the intercommunion has been ever becoming more frequent, more intricate, more widely ramified. There has all along been higher specialization, that there might be a larger generalization; and a deeper analysis, that there might be a better synthesis. Each larger generalization has lifted sundry specializations still higher; and each better synthesis has prepared the way for still deeper analysis.

And here we may fitly enter upon the task awhile since indicated—a sketch of the Genesis of Science, regarded as a gradual outgrowth from common knowledge—an extension of the perceptions by the aid of the reason. We propose to treat it as a psychological process historically displayed; tracing at the same time the advance from qualitative to quantitative prevision; the progress from concrete facts to abstract facts, and the application of such abstract facts to the analysis of new orders of concrete facts; the simultaneous advance in generalization and specialization; the continually increasing subdivision and reunion of the sciences; and their constantly improving *consensus*.

To trace out scientific evolution from its deepest roots would, of course, involve a complete analysis of the mind. For as science is a development of that common knowledge acquired by the unaided senses and uncultured reason, so is that common knowledge itself gradually built up out of the simplest perceptions. We must, therefore, begin somewhere abruptly; and the most appropriate stage to take for our point of departure will be the adult mind of the savage.

Commencing thus, without a proper preliminary analysis, we are naturally somewhat at a loss how to present, in a satisfactory manner, those fundamental processes of thought out of which science ultimately originates. Per-

haps our argument may be best initiated by the proposition, that all intelligent action whatever depends upon the discerning of distinctions among surrounding things. The condition under which only it is possible for any creature to obtain food and avoid danger is, that it shall be differently affected by different objects—that it shall be led to act in one way by one object, and in another way by another. In the lower orders of creatures this condition is fulfilled by means of an apparatus which acts automatically. In the higher orders the actions are partly automatic, partly conscious. And in man they are almost wholly conscious.

Throughout, however, there must necessarily exist a certain classification of things according to their properties—a classification which is either organically registered in the system, as in the inferior creation, or is formed by experience, as in ourselves. And it may be further remarked, that the extent to which this classification is carried, roughly indicates the height of intelligence—that, while the lowest organisms are able to do little more than discriminate organic from inorganic matter; while the generality of animals carry their classifications no further than to a limited number of plants or creatures serving for food, a limited number of beasts of prey, and a limited number of places and materials; the most degraded of the human race possess a knowledge of the distinctive natures of a great variety of substances, plants, animals, tools, persons, &c., not only as classes but as individuals.

What now is the mental process by which classification is effected? Manifestly it is a recognition of the *likeness* or *unlikeness* of things, either in respect of their sizes, colours, forms, weights, textures, tastes, &c., or in respect of their modes of action. By some special mark, sound, or motion, the savage identifies a certain four-legged creature he sees, as one that is good for food, and to be caught

in a particular way ; or as one that is dangerous ; and acts accordingly. He has classed together all the creatures that are *alike* in this particular. And manifestly in choosing the wood out of which to form his bow, the plant with which to poison his arrows, the bone from which to make his fish-hooks, he identifies them through their chief sensible properties as belonging to the general classes, wood, plant, and bone, but distinguishes them as belonging to sub-classes by virtue of certain properties in which they are *unlike* the rest of the general classes they belong to ; and so forms genera and species.

And here it becomes manifest that not only is classification carried on by grouping together in the mind things that are *like* ; but that classes and sub-classes are formed and arranged according to the *degrees of unlikeness*. Things widely contrasted are alone distinguished in the lower stages of mental evolution ; as may be any day observed in an infant. And gradually as the powers of discrimination increase, the widely contrasted classes at first distinguished, come to be each divided into sub-classes, differing from each other less than the classes differ ; and these sub-classes are again divided after the same manner. By the continuance of which process, things are gradually arranged into groups, the members of which are less and less *unlike* ; ending, finally, in groups whose members differ only as individuals, and not specifically. And thus there tends ultimately to arise the notion of *complete likeness*. For manifestly, it is impossible that groups should continue to be sub-divided in virtue of smaller and smaller differences, without there being a simultaneous approximation to the notion of *no difference*.

Let us next notice that the recognition of likeness and unlikeness, which underlies classification, and out of which continued classification evolves the idea of complete likeness—let us next notice that it also underlies the process

of *naming*, and by consequence *language*. For all language consists, at the beginning, of symbols which are as *like* to the things symbolized as it is practicable to make them. The language of signs is a means of conveying ideas by mimicking the actions or peculiarities of the things referred to. Verbal language is also, at the beginning, a mode of suggesting objects or acts by imitating the sounds which the objects make, or with which the acts are accompanied. Originally these two languages were used simultaneously. It needs but to watch the gesticulations with which the savage accompanies his speech—to see a Bushman or a Kaffir dramatizing before an audience his mode of catching game—or to note the extreme paucity of words in all primitive vocabularies; to infer that at first, attitudes, gestures, and sounds, were all combined to produce as good a *likeness* as possible, of the things, animals, persons, or events described; and that as the sounds came to be understood by themselves the gestures fell into disuse: leaving traces, however, in the manners of the more excitable civilized races. But be this as it may, it suffices simply to observe, how many of the words current among barbarous peoples are like the sounds appertaining to the things signified; how many of our own oldest and simplest words have the same peculiarity; how children tend to invent imitative words; and how the sign-language spontaneously formed by deaf mutes is invariably based upon imitative actions—to at once see that the notion of *likeness* is that from which the nomenclature of objects takes its rise.

Were there space we might go on to point out how this law of life is traceable, not only in the origin but in the development of language; how in primitive tongues the plural is made by a duplication of the singular, which is a multiplication of the word to make it *like* the multiplicity of the things; how the use of metaphor—that prolific

source of new words—is a suggesting of ideas that *are like* the ideas to be conveyed in some respect or other; and how, in the copious use of simile, fable, and allegory among uncivilized races, we see that complex conceptions, which there is yet no direct language for, are rendered, by presenting known conceptions more or less *like* them.

This view is further confirmed, and the predominance of this notion of likeness in primitive times further illustrated, by the fact that our system of presenting ideas to the eye originated after the same fashion. Writing and printing have descended from picture-language. The earliest mode of permanently registering a fact was by depicting it on a wall; that is—by exhibiting something as *like* to the thing to be remembered as it could be made. Gradually as the practice grew habitual and extensive, the most frequently repeated forms became fixed, and presently abbreviated; and, passing through the hieroglyphic and ideographic phases, the symbols lost all apparent relations to the things signified: just as the majority of our spoken words have done.

Observe again, that the same thing is true respecting the genesis of reasoning. The *likeness* that is perceived to exist between cases, is the essence of all early reasoning and of much of our present reasoning. The savage, having by experience discovered a relation between a certain object and a certain act, infers that the *like* relation will be found in future cases. And the expressions we constantly use in our arguments—“*analogy* implies,” “the cases are not *parallel*,” “by *parity* of reasoning,” “there is no *similarity*,”—show how constantly the idea of likeness underlies our ratiocinative processes.

Still more clearly will this be seen on recognising the fact that there is a certain parallelism between reasoning and classification; that the two have a common root; and that neither can go on without the other. For on the one

hand, it is a familiar truth that the attributing to a body in consequence of some of its properties, all those other properties in virtue of which it is referred to a particular class, is an act of inference. And, on the other hand, the forming of a generalization is the putting together in one class, all those cases which present like relations; while the drawing a deduction is essentially the perception that a particular case belongs to a certain class of cases previously generalized. So that as classification is a grouping together of *like things*; reasoning is a grouping together of *like relations* among things. Add to which, that while the perfection gradually achieved in classification consists in the formation of groups of *objects* which are *completely alike*; the perfection gradually achieved in reasoning consists in the formation of groups of *cases* which are *completely alike*.

Once more we may contemplate this dominant idea of likeness as exhibited in art. All art, civilized as well as savage, consists almost wholly in the making of objects *like* other objects; either as found in Nature, or as produced by previous art. If we trace back the varied art-products now existing, we find that at each stage the divergence from previous patterns is but small when compared with the agreement; and in the earliest art the persistency of imitation is yet more conspicuous. The old forms and ornaments and symbols were held sacred, and perpetually copied. Indeed, the strong imitative tendency notoriously displayed by the lowest human races, ensures among them a constant reproducing of likenesses of things, forms, signs, sounds, actions, and whatever else is imitable; and we may even suspect that this aboriginal peculiarity is in some way connected with the culture and development of this general conception, which we have found so deep and widespread in its applications.

And now let us go on to consider how, by a further unfolding of this same fundamental notion, there is a grad-

ual formation of the first germs of science. This idea of likeness which underlies classification, nomenclature, language spoken and written, reasoning, and art; and which plays so important a part because all acts of intelligence are made possible only by distinguishing among surrounding things, or grouping them into like and unlike;—this idea we shall find to be the one of which science is the especial product. Already during the stage we have been describing, there has existed *qualitative* prevision in respect to the commoner phenomena with which savage life is familiar; and we have now to inquire how the elements of *quantitative* prevision are evolved. We shall find that they originate by the perfecting of this same idea of likeness; that they have their rise in that conception of *complete likeness* which, as we have seen, necessarily results from the continued process of classification.

For when the process of classification has been carried as far as it is possible for the uncivilized to carry it—when the animal kingdom has been grouped not merely into quadrupeds, birds, fishes, and insects, but each of these divided into kinds—when there come to be sub-classes, in each of which the members differ only as individuals, and not specifically; it is clear that there must occur a frequent observation of objects which differ so little as to be indistinguishable. Among several creatures which the savage has killed and carried home, it must often happen that some one, which he wished to identify, is so exactly like another that he cannot tell which is which. Thus, then, there originates the notion of *equality*. The things which among ourselves are called *equal*—whether lines, angles, weights, temperatures, sounds or colours—are things which produce in us sensations that cannot be distinguished from each other. It is true that we now apply the word *equal* chiefly to the separate phenomena which objects exhibit, and not to groups of phenomena; but this limitation of the

idea has evidently arisen by subsequent analysis. And that the notion of equality did thus originate, will, we think, become obvious on remembering that as there were no artificial objects from which it could have been abstracted, it must have been abstracted from natural objects; and that the various families of the animal kingdom chiefly furnish those natural objects which display the requisite exactitude of likeness.

The same order of experiences out of which this general idea of equality is evolved, gives birth at the same time to a more complex idea of equality; or, rather, the process just described generates an idea of equality which further experience separates into two ideas—*equality of things* and *equality of relations*. While organic, and more especially animal forms, occasionally exhibit this perfection of likeness out of which the notion of simple equality arises, they more frequently exhibit only that kind of likeness which we call *similarity*; and which is really compound equality. For the similarity of two creatures of the same species but of different sizes, is of the same nature as the similarity of two geometrical figures. In either case, any two parts of the one bear the same ratio to one another, as the homologous parts of the other. Given in any species, the proportions found to exist among the bones, and we may, and zoologists do, predict from any one, the dimensions of the rest; just as, when knowing the proportions subsisting among the parts of a geometrical figure, we may, from the length of one, calculate the others. And if, in the case of similar geometrical figures, the similarity can be established only by proving exactness of proportion among the homologous parts; if we express this relation between two parts in the one, and the corresponding parts in the other, by the formula  $A$  is to  $B$  as  $a$  is to  $b$ ; if we otherwise write this,  $A$  to  $B = a$  to  $b$ ; if, consequently, the fact we prove is that the relation of  $A$  to  $B$  *equals* the relation of  $a$  to  $b$ ; then

it is manifest that the fundamental conception of similarity is *equality of relations*.

With this explanation we shall be understood when we say that the notion of equality of relations is the basis of all exact reasoning. Already it has been shown that reasoning in general is a recognition of *likeness* of relations; and here we further find that while the notion of likeness of things ultimately evolves the idea of simple equality, the notion of likeness of relations evolves the idea of equality of relations: of which the one is the concrete germ of exact science, while the other is its abstract germ.

Those who cannot understand how the recognition of similarity in creatures of the same kind, can have any alliance with reasoning, will get over the difficulty on remembering that the phenomena among which equality of relations is thus perceived, are phenomena of the same order and are present to the senses at the same time; while those among which developed reason perceives relations, are generally neither of the same order, nor simultaneously present. And if further, they will call to mind how Cuvier and Owen, from a single part of a creature, as a tooth, construct the rest by a process of reasoning based on this equality of relations, they will see that the two things are intimately connected, remote as they at first seem. But we anticipate. What it concerns us here to observe is, that from familiarity with organic forms there simultaneously arose the ideas of *simple equality*, and *equality of relations*.

At the same time, too, and out of the same mental processes, came the first distinct ideas of *number*. In the earliest stages, the presentation of several like objects produced merely an indefinite conception of multiplicity; as it still does among Australians, and Bushmen, and Damaras, when the number presented exceeds three or four. With such a fact before us we may safely infer that the first clear numerical conception was that of duality as contrasted with uni-

ty. And this notion of duality must necessarily have grown up side by side with those of likeness and equality ; seeing that it is impossible to recognise the likeness of two things without also perceiving that there are two. From the very beginning the conception of number must have been, as it is still, associated with the likeness or equality of the things numbered. If we analyze it, we find that simple enumeration is a registration of repeated impressions of any kind. That these may be capable of enumeration it is needful that they be more or less alike ; and before any *absolutely true* numerical results can be reached, it is requisite that the units be *absolutely equal*. The only way in which we can establish a numerical relationship between things that do not yield us like impressions, is to divide them into parts that *do* yield us like impressions. Two unlike magnitudes of extension, force, time, weight, or what not, can have their relative amounts estimated, only by means of some small unit that is contained many times in both ; and even if we finally write down the greater one as a unit and the other as a fraction of it, we state, in the denominator of the fraction, the number of parts into which the unit must be divided to be comparable with the fraction.

It is, indeed, true, that by an evidently modern process of abstraction, we occasionally apply numbers to unequal units, as the furniture at a sale or the various animals on a farm, simply as so many separate entities ; but no true result can be brought out by calculation with units of this order. And, indeed, it is the distinctive peculiarity of the calculus in general, that it proceeds on the hypothesis of that absolute equality of its abstract units, which no real units possess ; and that the exactness of its results holds only in virtue of this hypothesis. The first ideas of number must necessarily then have been derived from like or equal magnitudes as seen chiefly in organic objects ; and as the like

magnitudes most frequently observed were magnitudes of extension, it follows that geometry and arithmetic had a simultaneous origin.

Not only are the first distinct ideas of number co-ordinate with ideas of likeness and equality, but the first efforts at numeration displayed the same relationship. On reading the accounts of various savage tribes, we find that the method of counting by the fingers, still followed by many children, is the aboriginal method. Neglecting the several cases in which the ability to enumerate does not reach even to the number of fingers on one hand, there are many cases in which it does not extend beyond ten—the limit of the simple finger notation. The fact that in so many instances, remote, and seemingly unrelated nations, have adopted *ten* as their basic number; together with the fact that in the remaining instances the basic number is either *five* (the fingers of one hand) or *twenty* (the fingers and toes); almost of themselves show that the fingers were the original units of numeration. The still surviving use of the word *digit*, as the general name for a figure in arithmetic, is significant; and it is even said that our word *ten* (Sax. *tyñ*; Dutch, *tien*; German, *zehn*) means in its primitive expanded form *two hands*. So that originally, to say there were ten things, was to say there were two hands of them.

From all which evidence it is tolerably clear that the earliest mode of conveying the idea of any number of things, was by holding up as many fingers as there were things; that is—using a symbol which was *equal*, in respect of multiplicity, to the group symbolized. For which inference there is, indeed, strong confirmation in the recent statement that our own soldiers are even now spontaneously adopting this device in their dealings with the Turks. And here it should be remarked that in this recombination of the notion of equality with that of multiplicity, by which the first steps in numeration are effected, we may see one

of the earliest of those inoculations between the diverging branches of science, which are afterwards of perpetual occurrence.

Indeed, as this observation suggests, it will be well, before tracing the mode in which exact science finally emerges from the merely approximate judgments of the senses, and showing the non-serial evolution of its divisions, to note the non-serial character of those preliminary processes of which all after development is a continuation. On re-considering them it will be seen that not only are they divergent growths from a common root,—not only are they simultaneous in their progress; but that they are mutual aids; and that none can advance without the rest. That completeness of classification for which the unfolding of the perceptions paves the way, is impossible without a corresponding progress in language, by which greater varieties of objects are thinkable and expressible. On the one hand it is impossible to carry classification far without names by which to designate the classes; and on the other hand it is impossible to make language faster than things are classified.

Again, the multiplication of classes and the consequent narrowing of each class, itself involves a greater likeness among the things classed together; and the consequent approach towards the notion of complete likeness itself allows classification to be carried higher. Moreover, classification necessarily advances *pari passu* with rationality—the classification of *things* with the classification of *relations*. For things that belong to the same class are, by implication, things of which the properties and modes of behaviour—the co-existences and sequences—are more or less the same; and the recognition of this sameness of co-existences and sequences is reasoning. Whence it follows that the advance of classification is necessarily proportionate to the advance of generalizations. Yet further, the notion of *likeness*, both

in things and relations, simultaneously evolves by one process of culture the ideas of *equality* of things and *equality* of relations; which are the respective bases of exact concrete reasoning and exact abstract reasoning—Mathematics and Logic. And once more, this idea of equality, in the very process of being formed, necessarily gives origin to two series of relations—those of magnitude and those of number: from which arise geometry and the calculus. Thus the process throughout is one of perpetual subdivision and perpetual intercommunication of the divisions. From the very first there has been that *consensus* of different kinds of knowledge, answering to the *consensus* of the intellectual faculties, which, as already said, must exist among the sciences.

Let us now go on to observe how, out of the notions of *equality* and *number*, as arrived at in the manner described, there gradually arose the elements of quantitative prevision.

Equality, once having come to be definitely conceived, was readily applicable to other phenomena than those of magnitude. Being predicable of all things producing indistinguishable impressions, there naturally grew up ideas of equality in weights, sounds, colours, &c.; and indeed it can scarcely be doubted that the occasional experience of equal weights, sounds, and colours, had a share in developing the abstract conception of equality—that the ideas of equality in size, relations, forces, resistances, and sensible properties in general, were evolved during the same period. But however this may be, it is clear that as fast as the notion of equality gained definiteness, so fast did that lowest kind of quantitative prevision which is achieved without any instrumental aid, become possible.

The ability to estimate, however roughly, the amount of a foreseen result, implies the conception that it will be *equal to* a certain imagined quantity; and the correctness of the estimate will manifestly depend upon the accuracy at

which the perceptions of sensible equality have arrived. A savage with a piece of stone in his hand, and another piece lying before him of greater bulk but of the same kind (a fact which he infers from the *equality* of the two in colour and texture) knows about what effort he must put forth to raise this other piece; and he judges accurately in proportion to the accuracy with which he perceives that the one is twice, three times, four times, &c. as large as the other; that is—in proportion to the precision of his ideas of equality and number. And here let us not omit to notice that even in these vaguest of quantitative previsions, the conception of *equality of relations* is also involved. For it is only in virtue of an undefined perception that the relation between bulk and weight in the one stone is *equal* to the relation between bulk and weight in the other, that even the roughest approximation can be made.

But how came the transition from those uncertain perceptions of equality which the unaided senses give, to the certain ones with which science deals? It came by placing the things compared in juxtaposition. Equality being predicated of things which give us indistinguishable impressions, and no accurate comparison of impressions being possible unless they occur in immediate succession, it results that exactness of equality is ascertainable in proportion to the closeness of the compared things. Hence the fact that when we wish to judge of two shades of colour whether they are alike or not, we place them side by side; hence the fact that we cannot, with any precision, say which of two allied sounds is the louder, or the higher in pitch, unless we hear the one immediately after the other; hence the fact that to estimate the ratio of weights, we take one in each hand, that we may compare their pressures by rapidly alternating in thought from the one to the other; hence the fact, that in a piece of music, we can continue to make equal beats when the first beat has been given, but cannot

ensure commencing with the same length of beat on a future occasion; and hence, lastly, the fact, that of all magnitudes, those of *linear extension* are those of which the equality is most accurately ascertainable, and those to which by consequence all others have to be reduced. For it is the peculiarity of linear extension that it alone allows its magnitudes to be placed in *absolute* juxtaposition, or, rather, in coincident position; it alone can test the equality of two magnitudes by observing whether they will coalesce, as two equal mathematical lines do, when placed between the same points; it alone can test *equality* by trying whether it will become *identity*. Hence, then, the fact, that all exact science is reducible, by an ultimate analysis, to results measured in equal units of linear extension.

Still it remains to be noticed in what manner this determination of equality by comparison of linear magnitudes originated. Once more may we perceive that surrounding natural objects supplied the needful lessons. From the beginning there must have been a constant experience of like things placed side by side—men standing and walking together; animals from the same herd; fish from the same shoal. And the ceaseless repetition of these experiences could not fail to suggest the observation, that the nearer together any objects were, the more visible became any inequality between them. Hence the obvious device of putting in apposition, things of which it was desired to ascertain the relative magnitudes. Hence the idea of *measure*. And here we suddenly come upon a group of facts which afford a solid basis to the remainder of our argument; while they also furnish strong evidence in support of the foregoing speculations. Those who look sceptically on this attempted rehabilitation of the earliest epochs of mental development, and who more especially think that the derivation of so many primary notions from organic forms is somewhat strained, will perhaps see more probability in the several

hypotheses that have been ventured, on discovering that all measures of *extension* and *force* originated from the lengths and weights of organic bodies; and all measures of *time* from the periodic phenomena of either organic or inorganic bodies.

Thus, among linear measures, the cubit of the Hebrews was the *length of the forearm* from the elbow to the end of the middle finger; and the smaller scriptural dimensions are expressed in *hand-breadths* and *spans*. The Egyptian cubit, which was similarly derived, was divided into digits, which were *finger-breadths*; and each finger-breadth was more definitely expressed as being equal to four *grains of barley* placed breadthwise. Other ancient measures were the *orgyia* or *stretch of the arms*, the *pace*, and the *palm*. So persistent has been the use of these natural units of length in the East, that even now some of the Arabs mete out cloth by the forearm. So, too, is it with European measures. The *foot* prevails as a dimension throughout Europe, and has done since the time of the Romans, by whom, also, it was used: its lengths in different places varying not much more than men's feet vary. The heights of horses are still expressed in *hands*. The inch is the length of the terminal joint of *the thumb*; as is clearly shown in France, where *pouce* means both thumb and inch. Then we have the inch divided into three *barley-corns*.

So completely, indeed, have these organic dimensions served as the substrata of all mensuration, that it is only by means of them that we can form any estimate of some of the ancient distances. For example, the length of a degree on the Earth's surface, as determined by the Arabian astronomers shortly after the death of Haroun-al-Raschid, was fifty-six of their miles. We know nothing of their mile further than that it was 4000 cubits; and whether these were sacred cubits or common cubits, would remain doubtful, but that the length of the cubit is given as twen-

ty-seven inches, and each inch defined as the thickness of six barley-grains. Thus one of the earliest measurements of a degree comes down to us in barley-grains. Not only did organic lengths furnish those approximate measures which satisfied men's needs in ruder ages, but they furnished also the standard measures required in later times. One instance occurs in our own history. To remedy the irregularities then prevailing, Henry I. commanded that the ulna, or ancient ell, which answers to the modern yard, should be made of the exact length of *his own arm*.

Measures of weight again had a like derivation. Seeds seem commonly to have supplied the unit. The original of the carat used for weighing in India is *a small bean*. Our own systems, both troy and avoirdupois, are derived primarily from wheat-corns. Our smallest weight, the grain, is *a grain of wheat*. This is not a speculation; it is an historically registered fact. Henry III. enacted that an ounce should be the weight of 640 dry grains of wheat from the middle of the ear. And as all the other weights are multiples or sub-multiples of this, it follows that the grain of wheat is the basis of our scale. So natural is it to use organic bodies as weights, before artificial weights have been established, or where they are not to be had, that in some of the remoter parts of Ireland the people are said to be in the habit, even now, of putting a man into the scales to serve as a measure for heavy commodities.

Similarly with time. Astronomical periodicity, and the periodicity of animal and vegetable life, are simultaneously used in the first stages of progress for estimating epochs. The simplest unit of time, the day, nature supplies ready made. The next simplest period, the mooneth or month, is also thrust upon men's notice by the conspicuous changes constituting a lunation. For larger divisions than these,

the phenomena of the seasons, and the chief events from time to time occurring, have been used by early and uncivilized races. Among the Egyptians the rising of the Nile served as a mark. The New Zealanders were found to begin their year from the reappearance of the Pleiades above the sea. One of the uses ascribed to birds, by the Greeks, was to indicate the seasons by their migrations. Barrow describes the aboriginal Hottentot as denoting periods by the number of moons before or after the ripening of one of his chief articles of food. He further states that the Kaffir chronology is kept by the moon, and is registered by notches on sticks—the death of a favourite chief, or the gaining of a victory, serving for a new era. By which last fact, we are at once reminded that in early history, events are commonly recorded as occurring in certain reigns, and in certain years of certain reigns: a proceeding which practically made a king's reign a measure of duration.

And, as further illustrating the tendency to divide time by natural phenomena and natural events, it may be noticed that even by our own peasantry the definite divisions of months and years are but little used; and that they habitually refer to occurrences as “before sheep-shearing,” or “after harvest,” or “about the time when the squire died.” It is manifest, therefore, that the more or less equal periods perceived in Nature gave the first units of measure for time; as did Nature's more or less equal lengths and weights give the first units of measure for space and force.

It remains only to observe, as further illustrating the evolution of quantitative ideas after this manner, that measures of value were similarly derived. Barter, in one form or other, is found among all but the very lowest human races. It is obviously based upon the notion of *equality of worth*. And as it gradually merges into trade

by the introduction of some kind of currency, we find that the *measures of worth*, constituting this currency, are organic bodies; in some cases *cowries*, in others *cocoa-nuts*, in others *cattle*, in others *pigs*; among the American Indians peltry or *skins*, and in Iceland *dried fish*.

Notions of exact equality and of measure having been reached, there came to be definite ideas of relative magnitudes as being multiples one of another; whence the practice of measurement by direct apposition of a measure. The determination of linear extensions by this process can scarcely be called science, though it is a step towards it; but the determination of lengths of time by an analogous process may be considered as one of the earliest samples of quantitative prevision. For when it is first ascertained that the moon completes the cycle of her changes in about thirty days—a fact known to most uncivilized tribes that can count beyond the number of their fingers—it is manifest that it becomes possible to say in what number of days any specified phase of the moon will recur; and it is also manifest that this prevision is effected by an opposition of two times, after the same manner that linear space is measured by the opposition of two lines. For to express the moon's period in days, is to say how many of these units of measure are contained in the period to be measured—is to ascertain the distance between two points in time by means of a *scale of days*, just as we ascertain the distance between two points in space by a scale of feet or inches; and in each case the scale coincides with the thing measured—mentally in the one; visibly in the other. So that in this simplest, and perhaps earliest case of quantitative prevision, the phenomena are not only thrust daily upon men's notice, but Nature is, as it were, perpetually repeating that process of measurement by observing which the prevision is effected. And thus there may be signi-

fiance in the remark which some have made, that alike in Hebrew, Greek, and Latin, there is an affinity between the word meaning moon, and that meaning measure.

This fact, that in very early stages of social progress it is known that the moon goes through her changes in about thirty days, and that in about twelve moons the seasons return—this fact that chronological astronomy assumes a certain scientific character even before geometry does; while it is partly due to the circumstance that the astronomical divisions, day, month, and year, are ready-made for us, is partly due to the further circumstances that agricultural and other operations were at first regulated astronomically, and that from the supposed divine nature of the heavenly bodies their motions determined the periodical religious festivals. As instances of the one we have the observation of the Egyptians, that the rising of the Nile corresponded with the heliacal rising of Sirius; the directions given by Hesiod for reaping and ploughing, according to the positions of the Pleiades; and his maxim that “fifty days after the turning of the sun is a seasonable time for beginning a voyage.” As instances of the other, we have the naming of the days after the sun, moon, and planets; the early attempts among Eastern nations to regulate the calendar so that the gods might not be offended by the displacement of their sacrifices; and the fixing of the great annual festival of the Peruvians by the position of the sun. In all which facts we see that, at first, science was simply an appliance of religion and industry.

After the discoveries that a lunation occupies nearly thirty days, and that some twelve lunations occupy a year—discoveries of which there is no historical account, but which may be inferred as the earliest, from the fact that existing uncivilized races have made them—we come to the first known astronomical records, which are those of

eclipses. The Chaldeans were able to predict these. "This they did, probably," says Dr. Whewell in his useful history, from which most of the materials we are about to use will be drawn, "by means of their cycle of 223 months, or about eighteen years; for at the end of this time, the eclipses of the moon begin to return, at the same intervals and in the same order as at the beginning." Now this method of calculating eclipses by means of a recurring cycle,—the *Saros* as they called it—is a more complex case of prevision by means of coincidence of measures. For by what observations must the Chaldeans have discovered this cycle? Obviously, as Delambre infers, by inspecting their registers; by comparing the successive intervals; by finding that some of the intervals were alike; by seeing that these equal intervals were eighteen years apart; by discovering that *all* the intervals that were eighteen years apart were equal; by ascertaining that the intervals formed a series which repeated itself, so that if one of the cycles of intervals were superposed on another the divisions would fit. This once perceived, and it manifestly became possible to use the cycle as a scale of time by which to measure out future periods. Seeing thus that the process of so predicting eclipses, is in essence the same as that of predicting the moon's monthly changes by observing the number of days after which they repeat—seeing that the two differ only in the extent and irregularity of the intervals, it is not difficult to understand how such an amount of knowledge should so early have been reached. And we shall be less surprised, on remembering that the only things involved in these previsions were *time* and *number*; and that the time was in a manner self-numbered.

Still, the ability to predict events recurring only after so long a period as eighteen years, implies a considerable advance in civilization—a considerable development of general knowledge; and we have now to inquire what progress

in other sciences accompanied, and was necessary to, these astronomical previsions. In the first place, there must clearly have been a tolerably efficient system of calculation. Mere finger-counting, mere head-reckoning, even with the aid of a regular decimal notation, could not have sufficed for numbering the days in a year; much less the years, months, and days between eclipses. Consequently there must have been a mode of registering numbers; probably even a system of numerals. The earliest numerical records, if we may judge by the practices of the less civilized races now existing, were probably kept by notches cut on sticks, or strokes marked on walls; much as public-house scores are kept now. And there seems reason to believe that the first numerals used were simply groups of straight strokes, as some of the still-extant Roman ones are; leading us to suspect that these groups of strokes were used to represent groups of fingers, as the groups of fingers had been used to represent groups of objects—a supposition quite in conformity with the aboriginal system of picture writing and its subsequent modifications. Be this so or not, however, it is manifest that before the Chaldeans discovered their *Saros*, there must have been both a set of written symbols serving for an extensive numeration, and a familiarity with the simpler rules of arithmetic.

Not only must abstract mathematics have made some progress, but concrete mathematics also. It is scarcely possible that the buildings belonging to this era should have been laid out and erected without any knowledge of geometry. At any rate, there must have existed that elementary geometry which deals with direct measurement—with the apposition of lines; and it seems that only after the discovery of those simple proceedings, by which right angles are drawn, and relative positions fixed, could so regular an architecture be executed. In the case of the other division of concrete mathematics—mechanics, we have defi-

nite evidence of progress. We know that the lever and the inclined plane were employed during this period: implying that there was a qualitative prevision of their effects, though not a quantitative one. But we know more. We read of weights in the earliest records; and we find weights in ruins of the highest antiquity. Weights imply scales, of which we have also mention; and scales involve the primary theorem of mechanics in its least complicated form—involving not a qualitative but a quantitative prevision of mechanical effects. And here we may notice how mechanics, in common with the other exact sciences, took its rise from the simplest application of the idea of *equality*. For the mechanical proposition which the scales involve, is, that if a lever with *equal* arms, have *equal* weights suspended from them, the weights will remain at *equal* altitudes. And we may further notice, how, in this first step of rational mechanics, we see illustrated that truth awhile since referred to, that as magnitudes of linear extension are the only ones of which the equality is exactly ascertainable, the equalities of other magnitudes have at the outset to be determined by means of them. For the equality of the weights which balance each other in scales, wholly depends upon the equality of the arms: we can know that the weights are equal only by proving that the arms are equal. And when by this means we have obtained a system of weights,—a set of equal units of force, then does a science of mechanics become possible. Whence, indeed, it follows, that rational mechanics could not possibly have any other starting-point than the scales.

Let us further remember, that during this same period there was a limited knowledge of chemistry. The many arts which we know to have been carried on must have been impossible without a generalized experience of the modes in which certain bodies affect each other under special conditions. In metallurgy, which was extensively

practised, this is abundantly illustrated. And we even have evidence that in some cases the knowledge possessed was, in a sense, quantitative. For, as we find by analysis that the hard alloy of which the Egyptians made their cutting tools, was composed of copper and tin in fixed proportions, there must have been an established prevision that such an alloy was to be obtained only by mixing them in these proportions. It is true, this was but a simple empirical generalization; but so was the generalization respecting the recurrence of eclipses; so are the first generalizations of every science.

Respecting the simultaneous advance of the sciences during this early epoch, it only remains to remark that even the most complex of them must have made some progress—perhaps even a greater relative progress than any of the rest. For under what conditions only were the foregoing developments possible? There first required an established and organized social system. A long continued registry of eclipses; the building of palaces; the use of scales; the practice of metallurgy—alike imply a fixed and populous nation. The existence of such a nation not only presupposes laws, and some administration of justice, which we know existed, but it presupposes successful laws—laws conforming in some degree to the conditions of social stability—laws enacted because it was seen that the actions forbidden by them were dangerous to the State. We do not by any means say that all, or even the greater part, of the laws were of this nature; but we do say, that the fundamental ones were. It cannot be denied that the laws affecting life and property were such. It cannot be denied that, however little these were enforced between class and class, they were to a considerable extent enforced between members of the same class. It can scarcely be questioned, that the administration of them between members of the same class was seen by rulers to be necessary for keeping

their subjects together. And knowing, as we do, that other things equal, nations prosper in proportion to the justness of their arrangements, we may fairly infer that the very cause of the advance of these earliest nations out of aboriginal barbarism, was the greater recognition among them of the claims to life and property.

But supposition aside, it is clear that the habitual recognition of these claims in their laws, implied some prevision of social phenomena. Even thus early there was a certain amount of social science. Nay, it may even be shown that there was a vague recognition of that fundamental principle on which all the true social science is based—the equal rights of all to the free exercise of their faculties. That same idea of *equality*, which, as we have seen, underlies all other science, underlies also morals and sociology. The conception of justice, which is the primary one in morals; and the administration of justice, which is the vital condition of social existence; are impossible, without the recognition of a certain likeness in men's claims, in virtue of their common humanity. *Equity* literally means *equalness*; and if it be admitted that there were even the vaguest ideas of equity in these primitive eras, it must be admitted that there was some appreciation of the equalness of men's liberties to pursue the objects of life—some appreciation, therefore, of the essential principle of national equilibrium.

Thus in this initial stage of the positive sciences, before geometry had yet done more than evolve a few empirical rules—before mechanics had passed beyond its first theorem—before astronomy had advanced from its merely chronological phase into the geometrical; the most involved of the sciences had reached a certain degree of development—a development without which no progress in other sciences was possible.

Only noting as we pass, how, thus early, we may see that the progress of exact science was not only towards an

increasing number of previsions, but towards previsions more accurately quantitative—how, in astronomy, the recurring period of the moon's motions was by and by more correctly ascertained to be nineteen years, or two hundred and thirty-five lunations; how Callipus further corrected this Metonic cycle, by leaving out a day at the end of every seventy-six years; and how these successive advances implied a longer continued registry of observations, and the co-ordination of a greater number of facts—let us go on to inquire how geometrical astronomy took its rise.

The first astronomical instrument was the gnomon. This was not only early in use in the East, but it was found also among the Mexicans; the sole astronomical observations of the Peruvians were made by it; and we read that 1100 B.C., the Chinese found that, at a certain place, the length of the sun's shadow, at the summer solstice, was to the height of the gnomon, as one and a half to eight. Here again it is observable, not only that the instrument is found ready made, but that Nature is perpetually performing the process of measurement. Any fixed, erect object—a column, a dead palm, a pole, the angle of a building—serves for a gnomon; and it needs but to notice the changing position of the shadow it daily throws, to make the first step in geometrical astronomy. How small this first step was, may be seen in the fact that the only things ascertained at the outset were the periods of the summer and winter solstices, which corresponded with the least and greatest lengths of the mid-day shadow; and to fix which, it was needful merely to mark the point to which each day's shadow reached.

And now let it not be overlooked that in the observing at what time during the next year this extreme limit of the shadow was again reached, and in the inference that the sun had then arrived at the same turning point in his annual course, we have one of the simplest instances of that

combined use of *equal magnitudes* and *equal relations*, by which all exact science, all quantitative prevision, is reached. For the relation observed was between the length of the sun's shadow and his position in the heavens; and the inference drawn was that when, next year, the extremity of his shadow came to the same point, he occupied the same place. That is, the ideas involved were, the equality of the shadows, and the equality of the relations between shadow and sun in successive years. As in the case of the scales, the equality of relations here recognized is of the simplest order. It is not as those habitually dealt with in the higher kinds of scientific reasoning, which answer to the general type—the relation between two and three equals the relation between six and nine; but it follows the type—the relation between two and three, equals the relation between two and three; it is a case of not simply *equal* relations, but *coinciding* relations. And here, indeed, we may see beautifully illustrated how the idea of equal relations takes its rise after the same manner that that of equal magnitude does. As already shown, the idea of equal magnitudes arose from the observed coincidence of two lengths placed together; and in this case we have not only two coincident lengths of shadows, but two coincident relations between sun and shadows.

From the use of the gnomon there naturally grew up the conception of angular measurements; and with the advance of geometrical conceptions there came the hemisphere of Berosus, the equinoctial armil, the solstitial armil, and the quadrant of Ptolemy—all of them employing shadows as indices of the sun's position, but in combination with angular divisions. It is obviously out of the question for us here to trace these details of progress. It must suffice to remark that in all of them we may see that notion of equality of relations of a more complex kind, which is best illustrated in the astrolabe, an instrument which con-

sisted “ of circular rims, moveable one within the other, or about poles, and contained circles which were to be brought into the position of the ecliptic, and of a plane passing through the sun and the poles of the ecliptic”—an instrument, therefore, which represented, as by a model, the relative positions of certain imaginary lines and planes in the heavens; which was adjusted by putting these representative lines and planes into parallelism and coincidence with the celestial ones; and which depended for its use upon the perception that the relations between these representative lines and planes were *equal* to the relations between those represented.

Were there space, we might go on to point out how the conception of the heavens as a revolving hollow sphere, the discovery of the globular form of the earth, the explanation of the moon's phases, and indeed all the successive steps taken, involved this same mental process. But we must content ourselves with referring to the theory of eccentrics and epicycles, as a further marked illustration of it. As first suggested, and as proved by Hipparchus to afford an explanation of the leading irregularities in the celestial motions, this theory involved the perception that the progressions, retrogressions, and variations of velocity seen in the heavenly bodies, might be reconciled with their assumed uniform movement in circles, by supposing that the earth was not in the centre of their orbits; or by supposing that they revolved in circles whose centres revolved round the earth; or by both. The discovery that this would account for the appearances, was the discovery that in certain geometrical diagrams the relations were such, that the uniform motion of a point would, when looked at from a particular position, present analogous irregularities; and the calculations of Hipparchus involved the belief that the relations subsisting among these geometrical curves were *equal* to the relations subsisting among the celestial orbits.

Leaving here these details of astronomical progress, and the philosophy of it, let us observe how the relatively concrete science of geometrical astronomy, having been thus far helped forward by the development of geometry in general, reacted upon geometry, caused it also to advance, and was again assisted by it. Hipparchus, before making his solar and lunar tables, had to discover rules for calculating the relations between the sides and angles of triangles—*trigonometry* a subdivision of pure mathematics. Further, the reduction of the doctrine of the sphere to the quantitative form needed for astronomical purposes, required the formation of a *spherical trigonometry*, which was also achieved by Hipparchus. Thus both plane and spherical trigonometry, which are parts of the highly abstract and simple science of extension, remained undeveloped until the less abstract and more complex science of the celestial motions had need of them. The fact admitted by M. Comte, that since Descartes the progress of the abstract division of mathematics has been determined by that of the concrete division, is paralleled by the still more significant fact that even thus early the progress of mathematics was determined by that of astronomy.

And here, indeed, we may see exemplified the truth, which the subsequent history of science frequently illustrates, that before any more abstract division makes a further advance, some more concrete division must suggest the necessity for that advance—must present the new order of questions to be solved. Before astronomy presented Hipparchus with the problem of solar tables, there was nothing to raise the question of the relations between lines and angles; the subject-matter of trigonometry had not been conceived. And as there must be subject-matter before there can be investigation, it follows that the progress of the concrete divisions is as necessary to that of the abstract, as the progress of the abstract to that of the concrete.

Just incidentally noticing the circumstance that the epoch we are describing witnessed the evolution of algebra, a comparatively abstract division of mathematics, by the union of its less abstract divisions, geometry and arithmetic—a fact proved by the earliest extant samples of algebra, which are half algebraic, half geometric—we go on to observe that during the era in which mathematics and astronomy were thus advancing, rational mechanics made its second step; and something was done towards giving a quantitative form to hydrostatics, optics, and harmonics. In each case we shall see as before, how the idea of equality underlies all quantitative prevision; and in what simple forms this idea is first applied.

As already shown, the first theorem established in mechanics was, that equal weights suspended from a lever with equal arms would remain in equilibrium. Archimedes discovered that a lever with unequal arms was in equilibrium when one weight was to its arm as the other arm to its weight; that is—when the numerical relation between one weight and its arm was *equal* to the numerical relation between the other arm and its weight.

The first advance made in hydrostatics, which we also owe to Archimedes, was the discovery that fluids press *equally* in all directions; and from this followed the solution of the problem of floating bodies: namely, that they are in equilibrium when the upward and downward pressures are *equal*.

In optics, again, the Greeks found that the angle of incidence is *equal* to the angle of reflection; and their knowledge reached no further than to such simple deductions from this as their geometry sufficed for. In harmonics they ascertained the fact that three strings of *equal* lengths would yield the octave, fifth and fourth, when strained by weights having certain definite ratios; and they did not progress much beyond this. In the one of which cases we

see geometry used in elucidation of the laws of light; and in the other, geometry and arithmetic made to measure the phenomena of sound.

Did space permit, it would be desirable here to describe the state of the less advanced sciences—to point out how, while a few had thus reached the first stages of quantitative prevision, the rest were progressing in qualitative prevision—how some small generalizations were made respecting evaporation, and heat, and electricity, and magnetism, which, empirical as they were, did not in that respect differ from the first generalizations of every science—how the Greek physicians had made advances in physiology and pathology, which, considering the great imperfection of our present knowledge, are by no means to be despised—how zoology had been so far systematized by Aristotle, as, to some extent, enabled him from the presence of certain organs to predict the presence of others—how in Aristotle's *Politics*, there is some progress towards a scientific conception of social phenomena, and sundry previsions respecting them—and how in the state of the Greek societies, as well as in the writings of Greek philosophers, we may recognise not only an increasing clearness in that conception of equity on which the social science is based, but also some appreciation of the fact that social stability depends upon the maintenance of equitable regulations. We might dwell at length upon the causes which retarded the development of some of the sciences, as for example, chemistry; showing that relative complexity had nothing to do with it—that the oxidation of a piece of iron is a simpler phenomenon than the recurrence of eclipses, and the discovery of carbonic acid less difficult than that of the precession of the equinoxes—but that the relatively slow advance of chemical knowledge was due, partly to the fact that its phenomena were not daily thrust on men's notice as those of astronomy were; partly to the fact that Nature

does not habitually supply the means, and suggest the modes of investigation, as in the sciences dealing with time extension, and force; and partly to the fact that the great majority of the materials with which chemistry deals, instead of being ready to hand, are made known only by the arts in their slow growth; and partly to the fact that even when known, their chemical properties are not self-exhibited, but have to be sought out by experiment.

Merely indicating all these considerations, however, let us go on to contemplate the progress and mutual influence of the sciences in modern days; only parenthetically noticing how, on the revival of the scientific spirit, the successive stages achieved exhibit the dominance of the same law hitherto traced—how the primary idea in dynamics, a uniform force, was defined by Galileo to be a force which generates *equal* velocities in *equal* successive times—how the uniform action of gravity was first experimentally determined by showing that the time elapsing before a body thrown up, stopped, was *equal* to the time it took to fall—how the first fact in compound motion which Galileo ascertained was, that a body projected horizontally will have a uniform motion onwards and a uniformly accelerated motion downwards; that is, will describe *equal* horizontal spaces in *equal* times, compounded with *equal* vertical increments in *equal* times—how his discovery respecting the pendulum was, that its oscillations occupy *equal* intervals of time whatever their length—how the principle of virtual velocities which he established is, that in any machine the weights that balance each other, are reciprocally as their virtual velocities; that is, the relation of one set of weights to their velocities *equals* the relation of the other set of velocities to their weights;—and how thus his achievements consisted in showing the equalities of certain magnitudes and relations, whose equalities had not been previously recognised.

When mechanics had reached the point to which Galileo brought it—when the simple laws of force had been disentangled from the friction and atmospheric resistance by which all their earthly manifestations are disguised—when progressing knowledge of *physics* had given a due insight into these disturbing causes—when, by an effort of abstraction, it was perceived that all motion would be uniform and rectilinear unless interfered with by external forces—and when the various consequences of this perception had been worked out; then it became possible, by the union of geometry and mechanics, to initiate physical astronomy. Geometry and mechanics having diverged from a common root in men's sensible experiences; having, with occasional inoculations, been separately developed, the one partly in connexion with astronomy, the other solely by analyzing terrestrial movements; now join in the investigations of Newton to create a true theory of the celestial motions. And here, also, we have to notice the important fact that, in the very process of being brought jointly to bear upon astronomical problems, they are themselves raised to a higher phase of development. For it was in dealing with the questions raised by celestial dynamics that the then incipient infinitesimal calculus was unfolded by Newton and his continental successors; and it was from inquiries into the mechanics of the solar system that the general theorems of mechanics contained in the "Principia,"—many of them of purely terrestrial application—took their rise. Thus, as in the case of Hipparchus, the presentation of a new order of concrete facts to be analyzed, led to the discovery of new abstract facts; and these abstract facts having been laid hold of, gave means of access to endless groups of concrete facts before incapable of quantitative treatment.

Meanwhile, physics had been carrying further that progress without which, as just shown, rational mechanics

could not be disentangled. In hydrostatics, Stevinus had extended and applied the discovery of Archimedes. Torricelli had proved atmospheric pressure, "by showing that this pressure sustained different liquids at heights inversely proportional to their densities;" and Pascal "established the necessary diminution of this pressure at increasing heights in the atmosphere:" discoveries which in part reduced this branch of science to a quantitative form. Something had been done by Daniel Bernouilli towards the dynamics of fluids. The thermometer had been invented; and a number of small generalizations reached by it. Huyghens and Newton had made considerable progress in optics; Newton had approximately calculated the rate of transmission of sound; and the continental mathematicians had succeeded in determining some of the laws of sonorous vibrations. Magnetism and electricity had been considerably advanced by Gilbert. Chemistry had got as far as the mutual neutralization of acids and alkalies. And Leonardo da Vinci had advanced in geology to the conception of the deposition of marine strata as the origin of fossils. Our present purpose does not require that we should give particulars. All that it here concerns us to do is to illustrate the *consensus* subsisting in this stage of growth, and afterwards. Let us look at a few cases.

The theoretic law of the velocity of sound enunciated by Newton on purely mechanical considerations, was found wrong by one-sixth. The error remained unaccounted for until the time of Laplace, who, suspecting that the heat disengaged by the compression of the undulating strata of the air, gave additional elasticity, and so produced the difference, made the needful calculations and found he was right. Thus acoustics was arrested until thermology overtook and aided it. When Boyle and Marriot had discovered the relation between the density of gases and the pressures they are subject to; and when it thus became

possible to calculate the rate of decreasing density in the upper parts of the atmosphere; it also became possible to make approximate tables of the atmospheric refraction of light. Thus optics, and with it astronomy, advanced with barology. After the discovery of atmospheric pressure had led to the invention of the air-pump by Otto Guericke; and after it had become known that evaporation increases in rapidity as atmospheric pressure decreases; it became possible for Leslie, by evaporation in a vacuum, to produce the greatest cold known; and so to extend our knowledge of thermology by showing that there is no zero within reach of our researches. When Fourier had determined the laws of conduction of heat, and when the Earth's temperature had been found to increase below the surface one degree in every forty yards, there were data for inferring the past condition of our globe; the vast period it has taken to cool down to its present state; and the immense age of the solar system—a purely astronomical consideration.

Chemistry having advanced sufficiently to supply the needful materials, and a physiological experiment having furnished the requisite hint, there came the discovery of galvanic electricity. Galvanism reacting on chemistry disclosed the metallic bases of the alkalies, and inaugurated the electro-chemical theory; in the hands of Oersted and Ampère it led to the laws of magnetic action; and by its aid Faraday has detected significant facts relative to the constitution of light. Brewster's discoveries respecting double refraction and dipolarization proved the essential truth of the classification of crystalline forms according to the number of axes, by showing that the molecular constitution depends upon the axes. In these and in numerous other cases, the mutual influence of the sciences has been quite independent of any supposed hierarchical order. Often, too, their inter-actions are more complex than as

thus instanced—involve more sciences than two. One illustration of this must suffice. We quote it in full from the *History of the Inductive Sciences*. In Book XI., chap. II., on “The Progress of the Electrical Theory,” Dr. Whewell writes :—

“Thus at that period, mathematics was behind experiment, and a problem was proposed, in which theoretical results were wanted for comparison with observation, but could not be accurately obtained; as was the case in astronomy also, till the time of the approximate solution of the problem of three bodies, and the consequent formation of the tables of the moon and planets, on the theory of universal gravitation. After some time, electrical theory was relieved from this reproach, mainly in consequence of the progress which astronomy had occasioned in pure mathematics. About 1801 there appeared in the *Bulletin des Sciences*, an exact solution of the problem of the distribution of electric fluid on a spheroid, obtained by Biot, by the application of the peculiar methods which Laplace had invented for the problem of the figure of the planets. And, in 1811, M. Poisson applied Laplace’s artifices to the case of two spheres acting upon one another in contact, a case to which many of Coulomb’s experiments were referrible; and the agreement of the results of theory and observation, thus extricated from Coulomb’s numbers obtained above forty years previously, was very striking and convincing.”

Not only do the sciences affect each other after this direct manner, but they affect each other indirectly. Where there is no dependence, there is yet analogy—*equality of relations*; and the discovery of the relations subsisting among one set of phenomena, constantly suggests a search for the same relations among another set. Thus the established fact that the force of gravitation varies inversely as the square of the distance, being recognized as a necessary characteristic of all influences proceeding from a centre, raised the suspicion that heat and light follow the same law; which proved to be the case—a suspicion and a

confirmation which were repeated in respect to the electric and magnetic forces. Thus again the discovery of the polarization of light led to experiments which ended in the discovery of the polarization of heat—a discovery that could never have been made without the antecedent one. Thus, too, the known refrangibility of light and heat lately produced the inquiry whether sound also is not refrangible; which on trial it turns out to be.

In some cases, indeed, it is only by the aid of conceptions derived from one class of phenomena that hypotheses respecting other classes can be formed. The theory, at one time favoured, that evaporation is a solution of water in air, was an assumption that the relation between water and air is *like* the relation between salt and water; and could never have been conceived if the relation between salt and water had not been previously known. Similarly the received theory of evaporation—that it is a diffusion of the particles of the evaporating fluid in virtue of their atomic repulsion—could not have been entertained without a foregoing experience of magnetic and electric repulsions. So complete in recent days has become this *consensus* among the sciences, caused either by the natural entanglement of their phenomena, or by analogies in the relations of their phenomena, that scarcely any considerable discovery concerning one order of facts now takes place, without very shortly leading to discoveries concerning other orders.

To produce a tolerably complete conception of this process of scientific evolution, it would be needful to go back to the beginning, and trace in detail the growth of classifications and nomenclatures; and to show how, as subsidiary to science, they have acted upon it, and it has reacted upon them. We can only now remark that, on the one hand, classifications and nomenclatures have aided science by continually subdividing the subject-matter of research, and giv-

ing fixity and diffusion to the truths disclosed ; and that on the other hand, they have caught from it that increasing quantitateness, and that progress from considerations touching single phenomena to considerations touching the relations among many phenomena, which we have been describing.

Of this last influence a few illustrations must be given. In chemistry it is seen in the facts, that the dividing of matter into the four elements was ostensibly based upon the single property of weight ; that the first truly chemical division into acid and alkaline bodies, grouped together bodies which had not simply one property in common, but in which one property was constantly related to many others ; and that the classification now current, places together in groups *supporters of combustion, metallic and non-metallic bases, acids, salts, &c.*, bodies which are often quite unlike in sensible qualities, but which are like in the majority of their *relations* to other bodies. In mineralogy again, the first classifications were based upon differences in aspect, texture, and other physical attributes. Berzelius made two attempts at a classification based solely on chemical constitution. That now current, recognises as far as possible the *relations* between physical and chemical characters. In botany the earliest classes formed were *trees, shrubs, and herbs* : magnitude being the basis of distinction. Dioscorides divided vegetables into *aromatic, alimentary, medicinal, and vinous* : a division of chemical character. Cæsalpinus classified them by the seeds, and seed-vessels, which he preferred because of the *relations* found to subsist between the character of the fructification and the general character of the other parts.

While the "natural system" since developed, carrying out the doctrine of Linnæus, that "natural orders must be formed by attention not to one or two, but to *all* the parts of plants," bases its divisions on like peculiarities which are found

to be *constantly related* to the greatest number of other like peculiarities. And similarly in zoology, the successive classifications, from having been originally determined by external and often subordinate characters not indicative of the essential nature, have been gradually more and more determined by those internal and fundamental differences, which have uniform *relations* to the greatest number of other differences. Nor shall we be surprised at this analogy between the modes of progress of positive science and classification, when we bear in mind that both proceed by making generalizations; that both enable us to make previsions differing only in their precision; and that while the one deals with equal properties and relations, the other deals with properties and relations that approximate towards equality in variable degrees.

Without further argument, it will, we think, be sufficiently clear that the sciences are none of them separately evolved—are none of them independent either logically or historically; but that all of them have, in a greater or less degree, required aid and reciprocated it. Indeed, it needs but to throw aside theses, and contemplate the mixed character of surrounding phenomena, to at once see that these notions of division and succession in the kinds of knowledge are none of them actually true, but are simple scientific fictions—good, if regarded merely as aids to study; bad, if regarded as representing realities in Nature. Consider them critically, and no facts whatever are presented to our senses uncombined with other facts—no facts whatever but are in some degree disguised by accompanying facts: disguised in such a manner that all must be partially understood before any one can be understood. If it be said, as by M. Comte, that gravitating force should be treated of before other forces, seeing that all things are subject to it, it may on like grounds be said that heat should be first dealt with; seeing that thermal forces are everywhere in

action ; that the ability of any portion of matter to manifest visible gravitative phenomena depends on its state of aggregation, which is determined by heat ; that only by the aid of thermology can we explain those apparent exceptions to the gravitating tendency which are presented by steam and smoke, and so establish its universality, and that, indeed, the very existence of the solar system in a solid form is just as much a question of heat as it is one of gravitation.

Take other cases :—All phenomena recognised by the eyes, through which only are the data of exact science ascertainable, are complicated with optical phenomena ; and cannot be exhaustively known until optical principles are known. The burning of a candle cannot be explained without involving chemistry, mechanics, thermology. Every wind that blows is determined by influences partly solar, partly lunar, partly hygrometric ; and implies considerations of fluid equilibrium and physical geography. The direction, dip, and variations of the magnetic needle, are facts half terrestrial, half celestial—are caused by earthly forces which have cycles of change corresponding with astronomical periods. The flowing of the gulf-stream and the annual migration of icebergs towards the equator, depending as they do on the balancing of the centripetal and centrifugal forces acting on the ocean, involve in their explanation the Earth's rotation and spheroidal form, the laws of hydrostatics, the relative densities of cold and warm water, and the doctrines of evaporation. It is no doubt true, as M. Comte says, that “ our position in the solar system, and the motions, form, size, equilibrium of the mass of our world among the planets, must be known before we can understand the phenomena going on at its surface.” But, fatally for his hypothesis, it is also true that we must understand a great part of the phenomena going on at its surface before we can know its position, &c., in the solar system

It is not simply that, as we have already shown, those geometrical and mechanical principles by which celestial appearances are explained, were first generalized from terrestrial experiences; but it is that the very obtainment of correct data, on which to base astronomical generalizations, implies advanced terrestrial physics.

Until after optics had made considerable advance, the Copernican system remained but a speculation. A single modern observation on a star has to undergo a careful analysis by the combined aid of various sciences—has to be *digested by the organism of the sciences*; which have severally to assimilate their respective parts of the observation, before the essential fact it contains is available for the further development of astronomy. It has to be corrected not only for nutation of the earth's axis and for precession of the equinoxes, but for aberration and for refraction; and the formation of the tables by which refraction is calculated, presupposes knowledge of the law of decreasing density in the upper atmospheric strata; of the law of decreasing temperature, and the influence of this on the density; and of hygrometric laws as also affecting density. So that, to get materials for further advance, astronomy requires not only the indirect aid of the sciences which have presided over the making of its improved instruments, but the direct aid of an advanced optics, of barology, of thermology, of hygrometry; and if we remember that these delicate observations are in some cases registered electrically, and that they are further corrected for the "personal equation"—the time elapsing between seeing and registering, which varies with different observers—we may even add electricity and psychology. If, then, so apparently simple a thing as ascertaining the position of a star is complicated with so many phenomena, it is clear that this notion of the independence of the sciences, or certain of them, will not hold.

Whether objectively independent or not, they cannot

be subjectively so—they cannot have independence as presented to our consciousness; and this is the only kind of independence with which we are concerned. And here, before leaving these illustrations, and especially this last one, let us not omit to notice how clearly they exhibit that increasingly active *consensus* of the sciences which characterizes their advancing development. Besides finding that in these later times a discovery in one science commonly causes progress in others; besides finding that a great part of the questions with which modern science deals are so mixed as to require the co-operation of many sciences for their solution; we find in this last case that, to make a single good observation in the purest of the natural sciences, requires the combined assistance of half a dozen other sciences.

Perhaps the clearest comprehension of the interconnected growth of the sciences may be obtained by contemplating that of the arts, to which it is strictly analogous, and with which it is inseparably bound up. Most intelligent persons must have been, at one time or other, struck with the vast array of antecedents pre-supposed by one of our processes of manufacture. Let him trace the production of a printed cotton, and consider all that is implied by it. There are the many successive improvements through which the power-looms reached their present perfection; there is the steam-engine that drives them, having its long history from Papin downwards; there are the lathes in which its cylinder was bored, and the string of ancestral lathes from which those lathes proceeded; there is the steam-hammer under which its crank shaft was welded; there are the puddling-furnaces, the blast-furnaces, the coal-mines and the iron-mines needful for producing the raw material; there are the slowly improved appliances by which the factory was built, and lighted, and ventilated; there are the printing engine, and the die house, and the colour laboratory with its stock of materials from all parts of

the world, implying cochineal-culture, logwood-cutting, indigo-growing ; there are the implements used by the producers of cotton, the gins by which it is cleaned, the elaborate machines by which it is spun : there are the vessels in which cotton is imported, with the building-slips, the rope-yards, the sail-cloth factories, the anchor-forges, needful for making them ; and besides all these directly necessary antecedents, each of them involving many others, there are the institutions which have developed the requisite intelligence, the printing and publishing arrangements which have spread the necessary information, the social organization which has rendered possible such a complex co-operation of agencies.

Further analysis would show that the many arts thus concerned in the economical production of a child's frock, have each of them been brought to its present efficiency by slow steps which the other arts have aided ; and that from the beginning this reciprocity has been ever on the increase. It needs but on the one hand to consider how utterly impossible it is for the savage, even with ore and coal ready, to produce so simple a thing as an iron hatchet ; and then to consider, on the other hand, that it would have been impracticable among ourselves, even a century ago, to raise the tubes of the Britannia bridge from lack of the hydraulic press ; to at once see how mutually dependent are the arts, and how all must advance that each may advance. Well, the sciences are involved with each other in just the same manner. They are, in fact, inextricably woven into this same complex web of the arts ; and are only conventionally independent of it. Originally the two were one. How to fix the religious festivals ; when to sow : how to weigh commodities ; and in what manner to measure ground ; were the purely practical questions out of which arose astronomy, mechanics, geometry. Since then there has been a perpetual inosculation of the sciences and

the arts. Science has been supplying art with truer generalizations and more completely quantitative provisions. Art has been supplying science with better materials and more perfect instruments. And all along the interdependence has been growing closer, not only between art and science, but among the arts themselves, and among the sciences themselves.

How completely the analogy holds throughout, becomes yet clearer when we recognise the fact that *the sciences are arts to each other*. If, as occurs in almost every case, the fact to be analyzed by any science, has first to be prepared—to be disentangled from disturbing facts by the afore discovered methods of other sciences; the other sciences so used, stand in the position of arts. If, in solving a dynamical problem, a parallelogram is drawn, of which the sides and diagonal represent forces, and by putting magnitudes of extension for magnitudes of force a measurable relation is established between quantities not else to be dealt with; it may be fairly said that geometry plays towards mechanics much the same part that the fire of the founder plays towards the metal he is going to cast. If, in analyzing the phenomena of the coloured rings surrounding the point of contact between two lenses, a Newton ascertains by calculation the amount of certain interposed spaces, far too minute for actual measurement; he employs the science of number for essentially the same purpose as that for which the watchmaker employs tools. If, before writing down his observation on a star, the astronomer has to separate from it all the errors resulting from atmospheric and optical laws, it is manifest that the refraction-tables, and logarithm-books, and formulæ, which he successively uses, serve him much as retorts, and filters, and cupels serve the assayer who wishes to separate the pure gold from all accompanying ingredients.

So close, indeed, is the relationship, that it is impossible to say where science begins and art ends. All the in-

struments of the natural philosopher are the products of art; the adjusting one of them for use is an art; there is art in making an observation with one of them; it requires art properly to treat the facts ascertained; nay, even the employing established generalizations to open the way to new generalizations, may be considered as art. In each of these cases previously organized knowledge becomes the implement by which new knowledge is got at: and whether that previously organized knowledge is embodied in a tangible apparatus or in a formula, matters not in so far as its essential relation to the new knowledge is concerned. If, as no one will deny, art is applied knowledge, then such portion of a scientific investigation as consists of applied knowledge is art. So that we may even say that as soon as any prevision in science passes out of its originally passive state, and is employed for reaching other previsions, it passes from theory into practice—becomes science in action—becomes art. And when we thus see how purely conventional is the ordinary distinction, how impossible it is to make any real separation—when we see not only that science and art were originally one; that the arts have perpetually assisted each other; that there has been a constant reciprocation of aid between the sciences and arts; but that the sciences act as arts to each other, and that the established part of each science becomes an art to the growing part—when we recognize the closeness of these associations, we shall the more clearly perceive that as the connexion of the arts with each other has been ever becoming more intimate; as the help given by sciences to arts and by arts to sciences, has been age by age increasing; so the interdependence of the sciences themselves has been ever growing greater, their mutual relations more involved, their *consensus* more active.

In here ending our sketch of the Genesis of Science, we

are conscious of having done the subject but scant justice. Two difficulties have stood in our way: one, the having to touch on so many points in such small space; the other, the necessity of treating in serial arrangement a process which is not serial—a difficulty which must ever attend all attempts to delineate processes of development, whatever their special nature. Add to which, that to present in anything like completeness and proportion, even the outlines of so vast and complex a history, demands years of study. Nevertheless, we believe that the evidence which has been assigned suffices to substantiate the leading propositions with which we set out. Inquiry into the first stages of science confirms the conclusion which we drew from the analysis of science as now existing, that it is not distinct from common knowledge, but an outgrowth from it—an extension of the perception by means of the reason.

That which we further found by analysis to form the more specific characteristic of scientific previsions, as contrasted with the previsions of uncultured intelligence—their quantitateness—we also see to have been the characteristic alike in the initial steps in science, and of all the steps succeeding them. The facts and admissions cited in disproof of the assertion that the sciences follow one another, both logically and historically, in the order of their decreasing generality, have been enforced by the sundry instances we have met with, in which the more general or abstract sciences have been advanced only at the instigation of the more special or concrete—instances serving to show that a more general science as much owes its progress to the presentation of new problems by a more special science, as the more special science owes its progress to the solutions which the more general science is thus led to attempt—instances therefore illustrating the position that scientific advance is as much from the special to the general as from the general to the special.

Quite in harmony with this position we find to be the admissions that the sciences are as branches of one trunk, and that they were at first cultivated simultaneously; and this harmony becomes the more marked on finding, as we have done, not only that the sciences have a common root, but that science in general has a common root with language, classification, reasoning, art; that throughout civilization these have advanced together, acting and reacting upon each other just as the separate sciences have done; and that thus the development of intelligence in all its divisions and subdivisions has conformed to this same law which we have shown that the sciences conform to. From all which we may perceive that the sciences can with no greater propriety be arranged in a succession, than language, classification, reasoning, art, and science, can be arranged in a succession; that, however needful a succession may be for the convenience of books and catalogues, it must be recognized merely as a convention; and that so far from its being the function of a philosophy of the sciences to establish a hierarchy, it is its function to show that the linear arrangements required for literary purposes, have none of them any basis either in Nature or History.

There is one further remark we must not omit—a remark touching the importance of the question that has been discussed. Unfortunately it commonly happens that topics of this abstract nature are slighted as of no practical moment; and, we doubt not, that many will think it of very little consequence what theory respecting the genesis of science may be entertained. But the value of truths is often great, in proportion as their generality is wide. Remote as they seem from practical application, the highest generalizations are not unfrequently the most potent in their effects, in virtue of their influence on all those subordinate generalizations which regulate practice. And it must be so here. Whenever established, a correct theory of the

nistorical development of the sciences must have an immense effect upon education; and, through education, upon civilization. Greatly as we differ from him in other respects, we agree with M. Comte in the belief that, rightly conducted, the education of the individual must have a certain correspondence with the evolution of the race.

No one can contemplate the facts we have cited in illustration of the early stages of science, without recognising the *necessity* of the processes through which those stages were reached—a necessity which, in respect to the leading truths, may likewise be traced in all after stages. This necessity, originating in the very nature of the phenomena to be analyzed and the faculties to be employed, more or less fully applies to the mind of the child as to that of the savage. We say more or less fully, because the correspondence is not special but general only. Were the *environment* the same in both cases, the correspondence would be complete. But though the surrounding material out of which science is to be organized, is, in many cases, the same to the juvenile mind and the aboriginal mind, it is not so throughout; as, for instance, in the case of chemistry, the phenomena of which are accessible to the one, but were inaccessible to the other. Hence, in proportion as the environment differs, the course of evolution must differ. After admitting sundry exceptions, however, there remains a substantial parallelism; and, if so, it becomes of great moment to ascertain what really has been the process of scientific evolution. The establishment of an erroneous theory must be disastrous in its educational results; while the establishment of a true one must eventually be fertile in school-reforms and consequent social benefits.